Script **generated by TTT**

Title: Seidl: Virtual Machines (26.05.2014)

Date: Mon May 26 10:15:38 CEST 2014

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Pages: 45

[_ | a]

A program p is constructed as follows:

$$
\begin{array}{lcl}
t & ::= & a \mid X \mid \_ \mid f(t_1, \ldots, t_n) \\
g & ::= & p(t_1, \ldots, t_k) \mid X = t \\
c & ::= & p(X_1, \ldots, X_k) \leftarrow g_1, \ldots, g_r \\
p & ::= & c_1 \ldots . c_m ? g
\end{array}
$$

- A term t either is an atom, a variable, an anonymous variable or a constructor application.
- A goal g either is a literal, i.e., a predicate call, or a unification.
- A clause c consists of a head $p(X_1, \ldots, X_k)$ with predicate name and list of formal parameters together with a body, i.e., a sequence of goals.
- A program consists of a sequence of clauses together with a single goal as query.

A More Realistic Example:

$$
\begin{array}{lcl}
\mathsf{app}(X, Y, Z) & \leftarrow & X = [\,],\ Y = Z \\
\mathsf{app}(X, Y, Z) & \leftarrow & X = [H|X'],\ Z = [H|Z'],\ \mathsf{app}(X', Y, Z') \\
? & & \mathsf{app}(X, [Y, c], [a, b, Z])
\end{array}
$$

X = a Z = c
Y = b

Remark:

$[\,]$	$===$	the atom empty list
$[H\|Z]$	$===$	binary constructor application
$[a, b, Z]$	$===$	shortcut for: $[a\|[b\|[Z\|[\,]]]]$

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Procedural View of Proll programs:

literal	≡	procedure call
predicate	≡	procedure
clause	≡	definition
term	≡	value
unification	≡	basic computation step
binding of variables	≡	side effect

Note: Predicate calls ...

- ... do not have a return value.
- ... affect the caller through side effects only :-)
- ... may fail. Then the next definition is tried :-))

$\Longrightarrow$ backtracking

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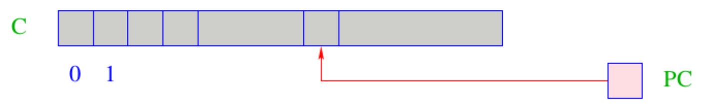
28 Architecture of the WiM:

The Code Store:

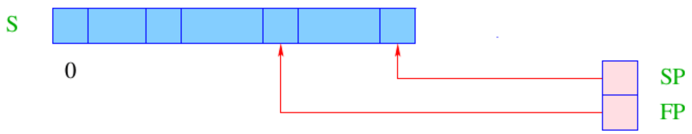


C = Code store – contains WiM program;
every cell contains one instruction;

PC = Program Counter – points to the next instruction to executed;

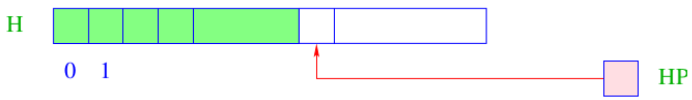
p(X) ← q(f(X)).

The Runtime Stack:



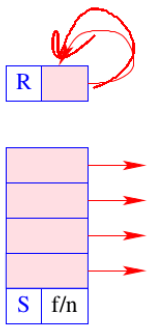
S	=	Runtime Stack – every cell may contain a value or an address;
SP	=	Stack Pointer – points to the topmost occupied cell;
FP	=	Frame Pointer – points to the current stack frame. Frames are created for predicate calls, contain cells for each variable of the current clause

The Heap:



H	=	Heap for dynamicly constructed terms;
HP	=	Heap-Pointer – points to the first free cell;

- The heap in maintained like a stack as well :-)
- A new-instruction allocates a object in H.
- Objects are tagged with their types (as in the MaMa) ...



A a	atom	1 cell
R	variable	1 cell
R	unbound variable	1 cell
S f/n	structure	(n+1) cells

29 Construction of Terms in the Heap

Parameter terms of goals (calls) are constructed in the heap before passing.

Assume that the address environment ρ returns, for each clause variable X its address (relative to FP) on the stack. Then $\text{code}_A\ t\ \rho$ should ...

- construct (a presentation of) t in the heap; and
- return a reference to it on top of the stack.

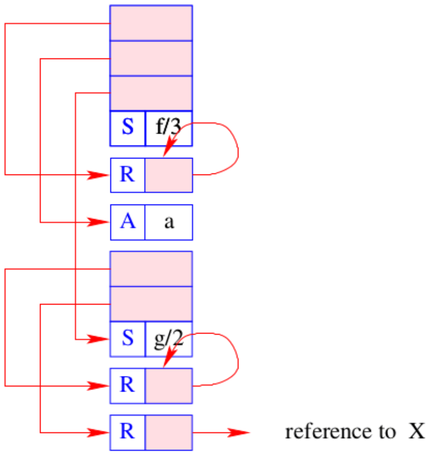
Idea:

- Construct the tree during a post-order traversal of t
- with one instruction for each new node!

Example: $t \equiv f(g(X, Y), a, Z)$.

Assume that X is initialized, i.e., $S[FP + \rho\, X]$ contains already a reference, Y and Z are not yet initialized.

Representing $t \equiv f(g(X,Y),a,Z)$:



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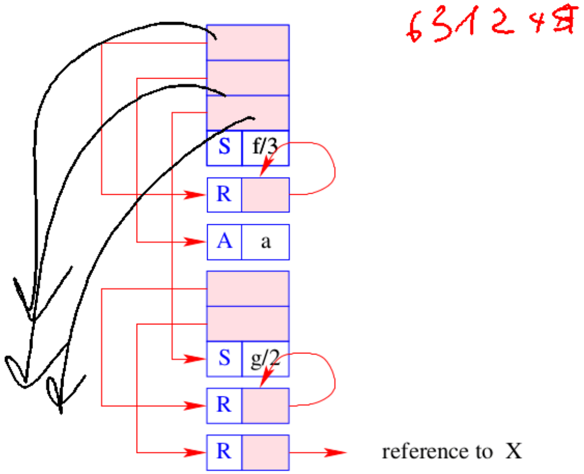

For a distinction, we mark occurrences of already initialized variables through over-lining (e.g. $\bar{X}$).

Note: Arguments are always initialized!

Then we define:

$$\begin{array}{lll} \text{code}_A\, a\, \rho & = & \text{putatom a} \\ \text{code}_A\, X\, \rho & = & \text{putvar}\, (\rho\, X) \\ \text{code}_A\, \bar{X}\, \rho & = & \text{putref}\, (\rho\, X) \\ \text{code}_A\, \_\, \rho & = & \text{putanon} \end{array} \qquad \begin{array}{lll} \text{code}_A\, f(t_1,\ldots,t_n)\, \rho & = & \text{code}_A\, t_1\, \rho \\ & & \ldots \\ & & \text{code}_A\, t_n\, \rho \\ & & \text{putstruct f/n} \end{array}$$

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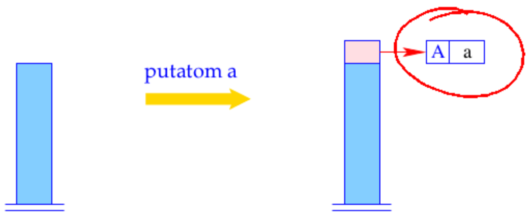
For $f(g(\overline{X},Y),a,Z)$ and $\rho = \{X \mapsto 1, Y \mapsto 2, Z \mapsto 3\}$ this results in the sequence:

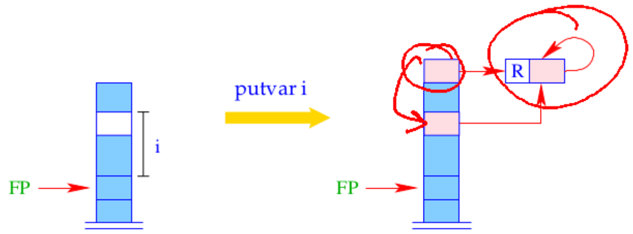
putref 1	putatom a
putvar 2	putvar 3
putstruct g/2	putstruct f/3

The instruction putatom a constructs an atom in the heap:



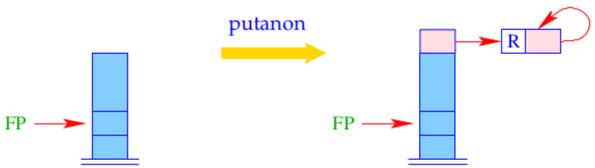
SP++; S[SP] = new (A,a);

The instruction putvar i introduces a new unbound variable and additionally initializes the corresponding cell in the stack frame:



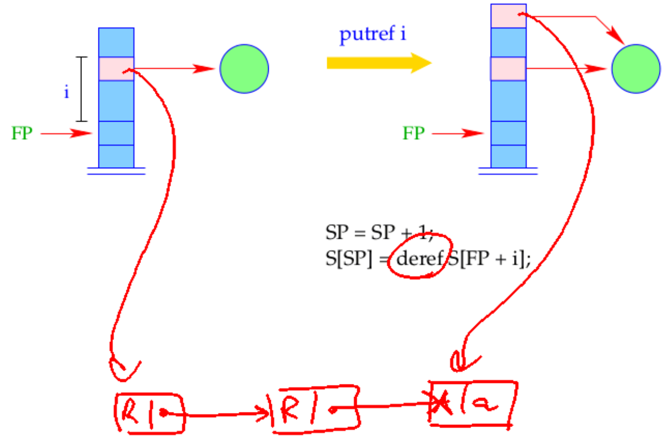
SP = SP + 1;
S[SP] = new (R, HP);
S[FP + i] = S[SP];

The instruction putanon introduces a new unbound variable but does not store a reference to it in the stack frame:



SP = SP + 1;
S[SP] = new (R, HP);

The instruction putref i pushes the value of the variable onto the stack:



SP = SP + 1;
S[SP] = deref S[FP + i];

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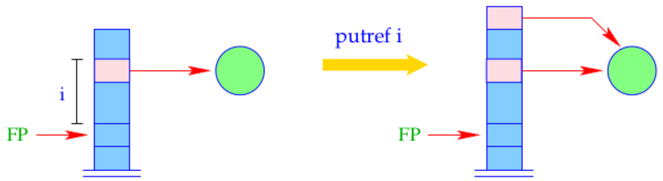
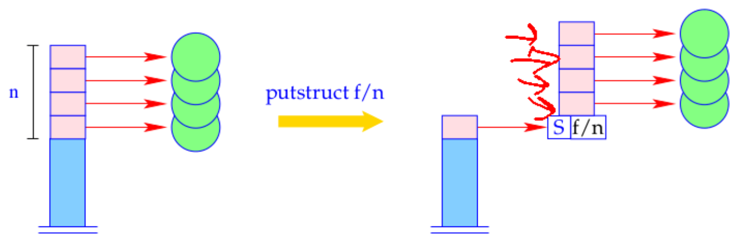


SP = SP + 1;
S[SP] = deref S[FP + i];

The auxiliary function deref contracts chains of references:

```
ref deref (ref v) {
     if (H[v]==(R,w) && v!=w) return deref (w);
     else return v;
}
```


The instruction putstruct f/n builds a constructor application in the heap:



v = new (S, f, n);
SP = SP - n + 1;
for (i=1; i<=n; i++)
  H[v + i] = S[SP + i -1];
S[SP] = v;

Remarks:

- The instruction putref i does not just push the reference from $S[FP + i]$ onto the stack, but also dereferences it as much as possible

  $\Longrightarrow$ maximal contraction of reference chains.

- In constructed terms, references always point to smaller heap addresses. Also otherwise, this will be often the case. Sadly enough, it cannot be guaranteed in general :-(

30 The Translation of Literals (Goals)

Idea:

- Literals are treated as procedure calls.
- We first allocate a stack frame.
- Then we construct the actual parameters (in the heap)
- ... and store references to these into the stack frame.
- Finally, we jump to the code for the procedure/predicate.

$$\text{code}_G\ p(t_1, \ldots, t_k)\ \rho \quad = \quad \begin{array}{ll} \text{mark B} & \text{// allocates the stack frame} \\ \text{code}_A\ t_1\ \rho & \\ \ldots & \\ \text{code}_A\ t_k\ \rho & \\ \text{call p/k} & \text{// calls the procedure p/k} \\ \text{B : } \ldots & \end{array}$$

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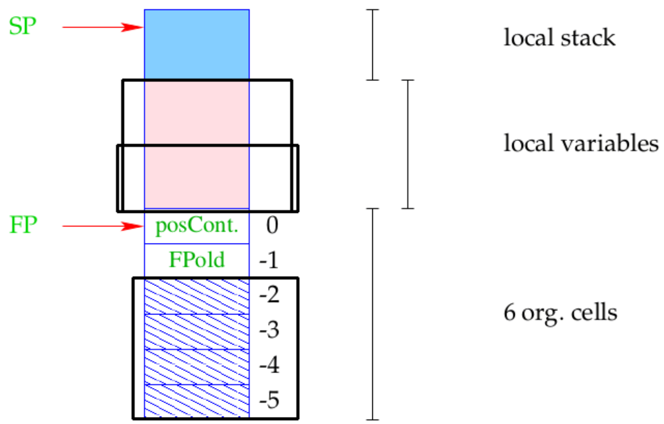
Example: $p(a, X, g(\bar{X}, Y))$ with $\rho = \{X \mapsto 1, Y \mapsto 2\}$

We obtain:

mark B	putref 1		call p/3
putatom a	putvar 2	B:	...
putvar 1	putstruct g/2		

Stack Frame of the WiM:

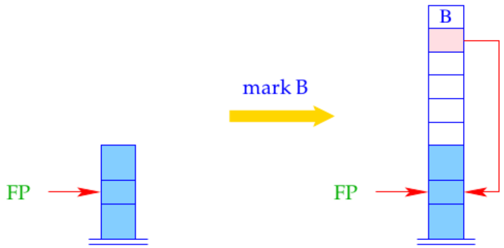


Remarks:

- The positive continuation address records where to continue after successful treatment of the goal.
- Additional organizational cells are needed for the implementation of backtracking

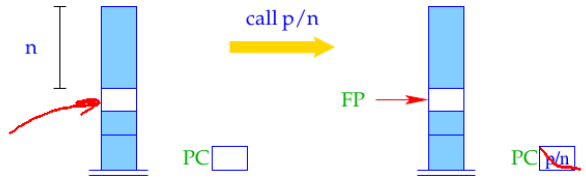
  $\Longrightarrow$ will be discussed at the translation of predicates.

The instruction mark B allocates a new stack frame:



SP = SP + 6;
S[SP] = B; S[SP-1] = FP;

The instruction call p/n calls the n-ary predicate p :



FP = SP - n;
PC = p/n;

31 Unification

Convention:

- By $\tilde{X}$, we denote an occurrence of X;
  it will be translated differently depending on whether the variable is initialized or not.
- We introduce the macro put $\tilde{X}$ ρ :

$$\begin{array}{lll} \text{put } X\ \rho & = & \text{putvar } (\rho\ X) \\ \text{put } \_\ \rho & = & \text{putanon} \\ \text{put } \bar{X}\ \rho & = & \text{putref } (\rho\ X) \end{array}$$

Let us translate the unification $\tilde{X} = t$.

Idea 1:

- Push a reference to (the binding of) X onto the stack;
- Construct the term t in the heap;
- Invent a new instruction implementing the unification :-)

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$$
\begin{aligned}
\text{code}_G\ (\tilde{X} = t)\ \rho \quad = \quad & \text{put}\ \tilde{X}\ \rho \\
& \text{code}_A\ t\ \rho \\
& \text{unify}
\end{aligned}
$$

Example:

Consider the equation:

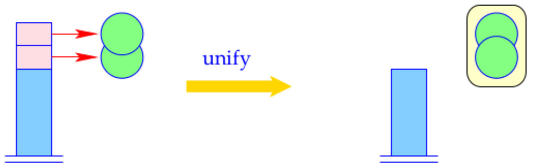
$$\bar{U} = f(g(\bar{X}, Y), a, Z)$$

Then we obtain for an address environment

$$\rho = \{X \mapsto 1, Y \mapsto 2, Z \mapsto 3, U \mapsto 4\}$$

putref 4	putref 1	putatom a	unify
	putvar 2	putvar 3	
	putstruct g/2	putstruct f/3	

The instruction unify calls the run-time function `unify()` for the topmost two references:



unify (S[SP-1], S[SP]);
SP = SP–2;

The Function `unify()`

- ... takes two heap addresses.
  For each call, we guarantee that these are maximally de-referenced.
- ... checks whether the two addresses are already identical.
  If so, does nothing :-)
- ... binds younger variables (larger addresses) to older variables (smaller addresses);
- ... when binding a variable to a term, checks whether the variable occurs inside the term $\Longrightarrow$ occur-check;
- ... records newly created bindings;
- ... may fail. Then backtracking is initiated.

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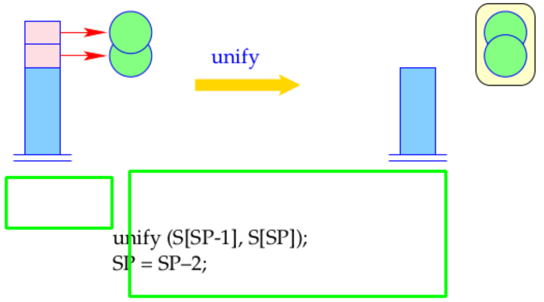

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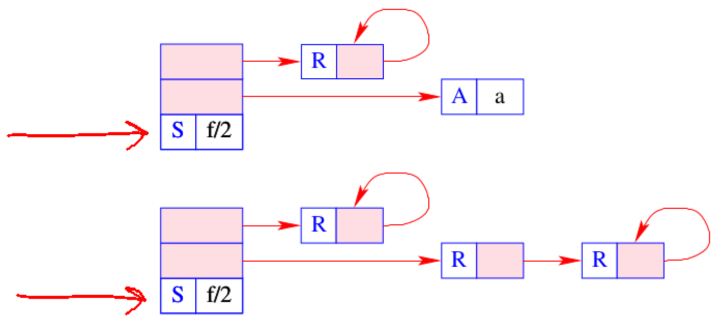
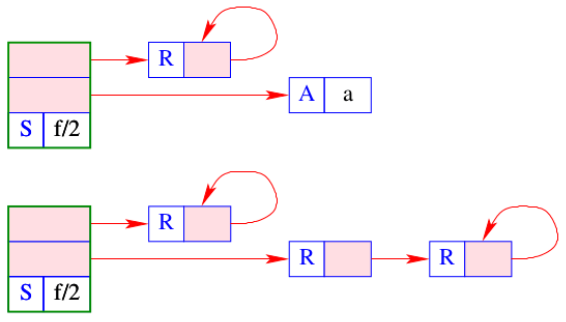
```
bool unify (ref u, ref v) {
   if (u == v) return true;
   if (H[u] == (R,_)) {
      if (H[v] == (R,_)) {
         if (u>v) {
            H[u] = (R,v); trail (u); return true;
         } else {
            H[v] = (R,u); trail (v); return true;
         }
      } elseif (check (u,v)) {
         H[u] = (R,v); trail (u); return true;
      } else {
         backtrack(); return false;
      }
   }
   ...
```



```
...
 if ((H[v] == (R,_)) {
      if (check (v,u)) {
         H[v] = (R,u); trail (v); return true;
      } else {
         backtrack(); return false;
      }
   }
   if (H[u]==(A,a) && H[v]==(A,a))
      return true;
   if (H[u]==(S, f/n) && H[v]==(S, f/n)) {
      for (int i=1; i<=n; i++)
         if(!unify (deref (H[u+i]), deref (H[v+i])) return false;
      return true;
   }
   backtrack(); return false;
}
```


R
A a
S f/2
R
R R
S f/2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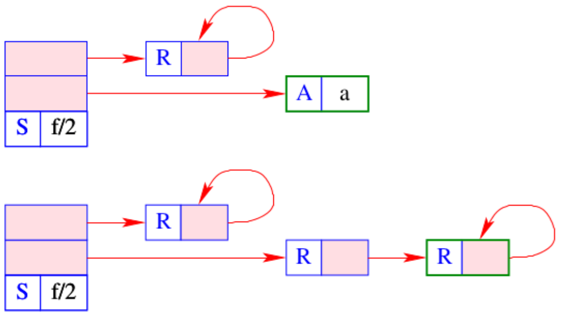
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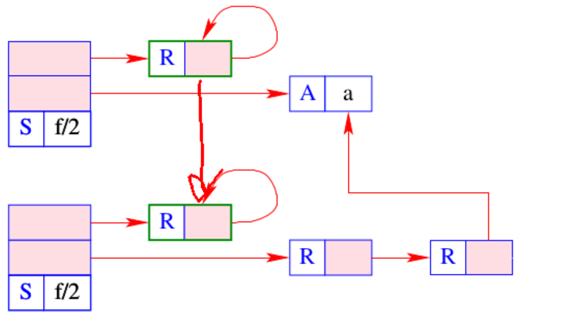
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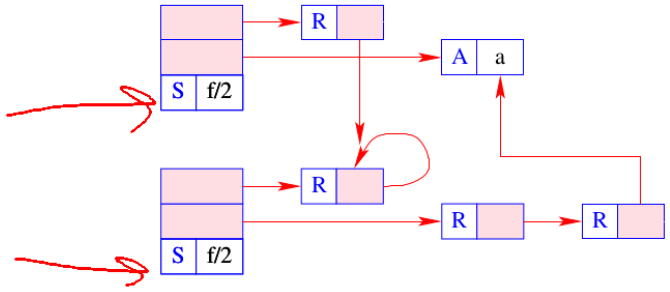

- The run-time function `trail()` records the a potential new binding.
- The run-time function `backtrack()` initiates backtracking.
- The auxiliary function `check()` performs the occur-check: it tests whether a variable (the first argument) occurs inside a term (the second argument).
- Often, this check is skipped, i.e.,

```
bool check (ref u, ref v) { return true;}
```