



Script generated by TTT

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## Social Gaming / Social Computing SS 2015

PD Dr. Georg Groh



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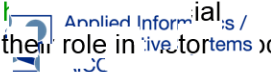
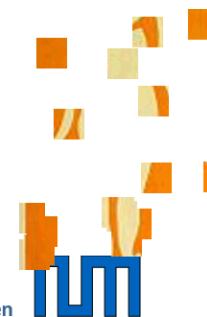
Working menu, target groups for Georg Groh



## Social Gaming / Social Computing SS 2015

Working menu, target groups for Georg Groh

- ethnology (ethnic groups)
- ethnology & their role in live (artistic) societies
- ethnology, e.g. artists' groups (Bauhaus, Brücke, Die Brücke) with distinct philosophy, manifested in their work (artistic frame)





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= Tönnis (1885-1936):

## Social anthropology, social computing SS 2015


Small group research (see [4,5,6])

Working conditions, target groups for Georg Groh

ethnology (ethnic groups)

groups & their role in life, totems societies

groups (Bauhaus, Brücke, realists) with distinct philosophy, manifested in social frame), not a




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Tönnis (1884-1935):

# Social-Action Theory and Group Theory SS 2015

1930s: Small group netsearch (see [4, 5, 6])


ist: com noi, comm... , comm...  
 - fivi... al ro... no... group a... est...  
 tur... frie... groups f... Georg Groh  
 menu, g... (e.g. political party)

llex in (e.g. family) ... group

- ethnology (ethnic group)
- eth ("my group") (special in group s...

& their r... in... groups  
 & their r... in... groups

Homans (1950) [6]: "A group is a nu...  
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### Social Networks: cliques vs. communities

Definition: A **clique** is a group of nodes in a network where every node is connected to every other node in the group (complete subgraph).

Definition: A **community** is a group of nodes that are more densely connected to each other than to nodes outside the group.

Example: In a social network, a group of friends who all know each other is a clique. A group of people who mostly know each other but also have connections to people outside the group is a community.

Clustering Coefficient (C): A measure of the degree to which nodes in a network tend to cluster together. It is calculated as the ratio of the number of triangles in the network to the number of possible triangles.

Community Structure: A network with a high clustering coefficient and a low average path length is said to have a "small-world" structure, which is characteristic of many real-world networks.

Applications: Clustering algorithms are used to identify communities in social networks, biological networks, and other complex systems.



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Tönnis (18... [3]:

work w.

SS 15

compactness: mean average lengths are small and/or connectivity is high (communities (e.g. political party))


intra cluster coherence

eth. network analysis

group members have more ties with each other

Homage → "inter cluster coherence"

and are independent: Moon [12]. Each member is at least 100m from other members → distance between members



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$\delta(G(I))$  is  $\frac{1}{|I|} \sum_{v \in I} \deg(v)$

$\delta(G(I)) = \frac{1}{|I|} \sum_{v \in I} \deg(v) \geq \frac{1}{|I|} \sum_{v \in I} \frac{2|E|}{|V|} = \frac{2|E|}{|V|} \frac{|I|}{|V|}$

Ist  $\dots$

- **completely connected**: if  $|I|=k$  in  $G$ , then  $G[I]$  is  $(k-1)$ -vertex-edge-connected
- **intracohere**: group members have many edges between them
- **eth.**: ("try arc...")
- **& they are u...**

**Hom**  $G$  is  $n$ -vertex connected if  $|V| > n$  and  $G - X$  is connected for  $|X| < n$

• **over is rasy**: "disconnected" if  $|V| > 2$  and  $G - Y$  is c...

• **at least** ...  $|E|$  with  $|Y| < n$ ; distance bet...



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SS 2015

$\text{diam}(G(U)) \leq N$   
 is **N-clique** iff  $\forall u, v \in U : \text{dist}_G(u, v) \leq N$

is **N-clan** iff  $\forall u, v \in U : \text{dist}_G(u, v) \leq N$  and  $U$  is maximal

& then  $G$  is **n-vertex connected** if  $|V| > n$  and  $G - X$  is connected for  $|X| < n$

Homomorphism  $f: V \rightarrow V$  with  $|X| < n$

at least  $\lfloor n/2 \rfloor$

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Find (just drop all  $n$ -cliques)

$U$  is  $N$ -clique iff  $U$  is maximal  $N$ -clique

& then all  $N$ -cliques;

Homomorphisms contained within  $G$

can have  $N$ -cliques a  $N$ -clique

that maximal  $N$ -cliques:  $\{1, 2, 3, 4\}$

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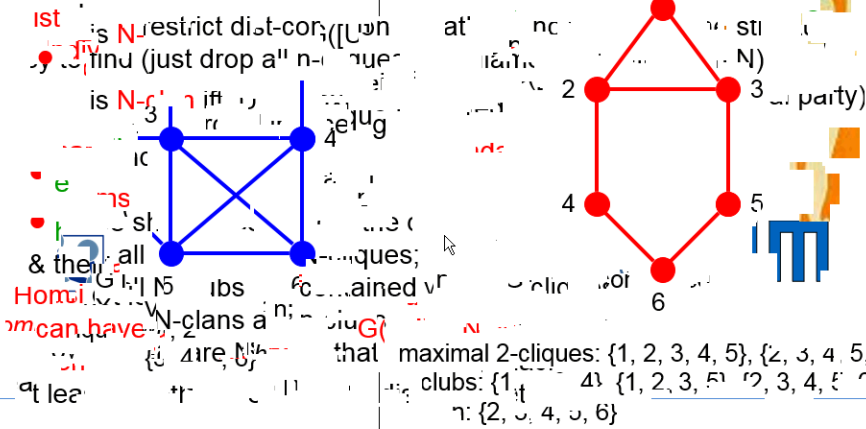
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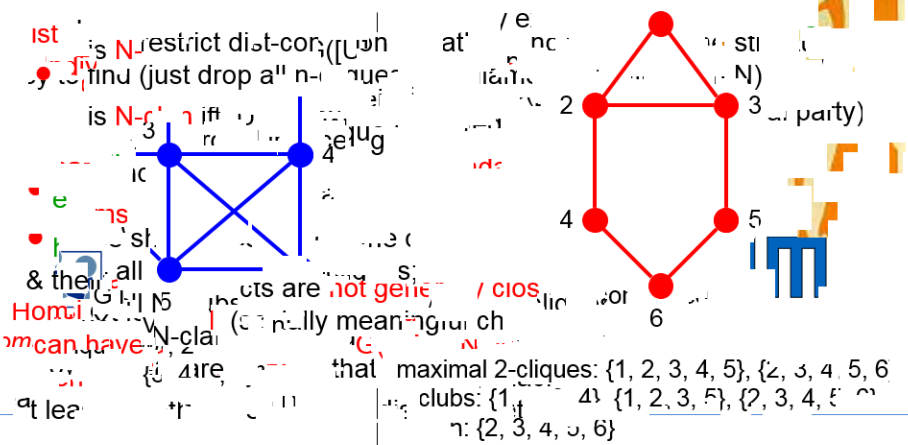
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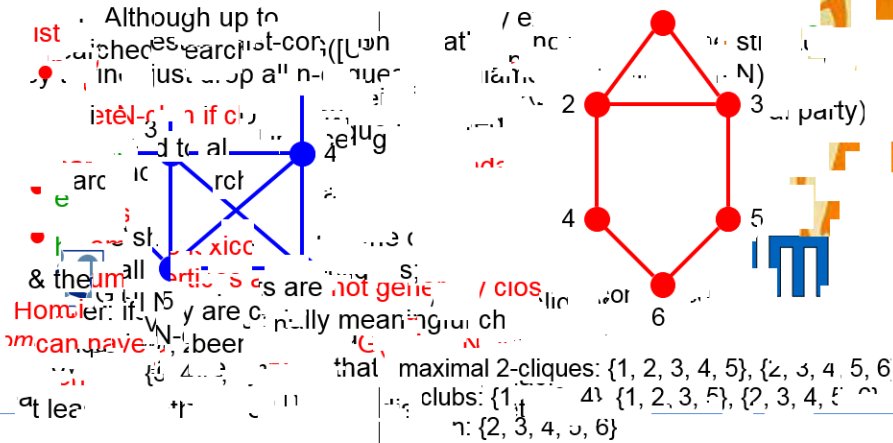
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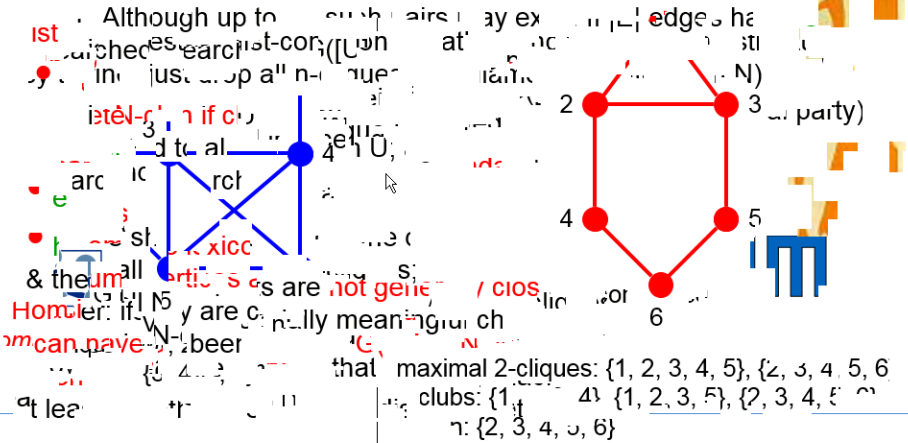
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$\text{diam}(G(U)) \leq N$  iff  $\text{diam}(G(U)) \leq N$

Determine if  $U$  is a clique.

Although up to  $N^2$  edges have to be checked, search for  $U$  in  $\mathcal{C}(G)$  is just a loop all neighbors.

Compute lexicographically smallest  $U$  and the minimum vertices are  $\{1, 2, 3, 4, 5, 6\}$ .

Homomorphism: if  $N$  is large, it is not generally meaningful.

Maximal 2-cliques:  $\{1, 2, 3, 4, 5\}, \{2, 3, 4, 5, 6\}$   
 Clubs:  $\{1, 2, 3, 4, 5\}, \{2, 3, 4, 5, 6\}$   
 At least:  $\{2, 3, 4, 5, 6\}$

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Maximal 2-cliques:  $\{1, 2, 3, 4, 5\}, \{2, 3, 4, 5, 6\}$   
 Clubs:  $\{1, 2, 3, 4, 5\}, \{2, 3, 4, 5, 6\}$   
 At least:  $\{2, 3, 4, 5, 6\}$

$\text{list}_G(u, v) \leq N$  Exhaustive search:  $O(|E| + |V|)$   
 $\text{list}_G(u, v) \leq N$  Exhaustive search:  $O(n^2)$   
 $\text{list}_G(u, v) \leq N$  Exhaustive search:  $O(|V|^2 2^M)$

There are a substantial number of maximal 2-cliques in a graph.

Compute lexicographically smallest maximal 2-clique & the union of all maximal 2-cliques is the set of all vertices in  $V-U$  in ascending order.

Homoser: if  $N$  is large, it is not possible to list all maximal 2-cliques.

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$\text{blemas: club iff } \text{diam} \leq N$   
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 ere a sub... better...  
 ist... search...  
 In... decision...  
 with... k nodes...  
 arc...  
 compute lexicographically small...  
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 can pave... beer...  
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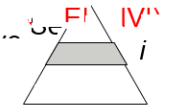
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..., n levels, leaves only / rel n

ch level: vertex  $v_i$

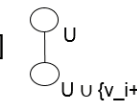

Search for maximal cliques in  $G[v_1, v_2, \dots, v_i]$



level  $i+1$ : determine children of node  $U$  at level  $i$ : two cases:

- 1)  $v_{i+1}$  adjacent to all nodes  $J \subseteq N(v_i)$ 
  - $\rightarrow U \cup \{v_{i+1}\}$  is maximal clique in  $G[v_1, v_2, \dots, v_{i+1}]$
  - with  $v_{i+1}$  is only child of  $U$
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  - & the remaining vertices are sorted; list all potential children of  $U$
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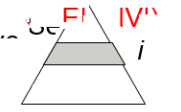
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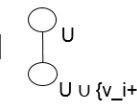
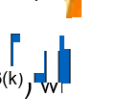
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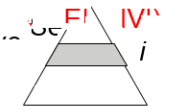
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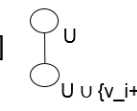

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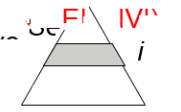
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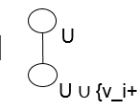

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Algorithm: traverse and find all maximal cliques in  $G$

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  - $\rightarrow$  if  $(U - N(v_{i+1})) \cup \{v_{i+1}\}$  is non-empty, it is also a maximal clique in  $G[v_1, v_2, \dots, v_{i+1}]$
  - & the children are sorted, Test  $v_{i+1}$  vertex
  - Homter  $(U - N(v_{i+1})) \cup \{v_{i+1}\}$  will be a child of clique
  - possible nodes  $\rightarrow$  can pay beer checked search is over
  - lexigraphically

at leaf

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Search for maximal cliques in  $G[v_1, v_2, \dots, v_i]$

level  $i+1$ : children of node  $U$  at level  $i$ : two cases:

Parent( $U, i$ ): first node  $U$  at level  $i$ :

- smallest clique in  $G[v_1, v_2, \dots, v_i]$  is  $U$ 
  - $U$  is maximal clique in  $G[v_1, v_2, \dots, v_{i+1}]$
  - with  $v_{i+1}$  is only child of  $U$
- Rig vertex in  $U$  not adjacent to  $v_{i+1}$ 
  - if case (1): no right child
  - if case (2): if  $X = (U - N(v_{i+1})) \cup \{v_{i+1}\}$  is maximal
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Algorithm: traverse and find out tree structure (r, heavy)

Step 1:  $\text{Parent}(U, i)$  for node  $U$  at level  $i$ : two cases:

- Case 1:  $\text{Parent}(U, i) = U$  if  $i = 0$
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Step 2:  $\text{Rig}(v_i, \text{hilo}(i))$ : adjacent to  $U_{i-1}$  (level  $i-1$ )

Step 3:  $\text{effC}(U, i)$ : either  $U$  or  $U_{i-1}$

Step 4:  $\text{Homo}(U, i)$ :  $U$  and  $U_{i-1}$  are leaves iff  $\delta(G([U])) \geq \dots$

Step 5:  $\text{at leaf}$

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ins \_C\_ \_S

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... are generally not r ...

... Rig ...

→ If an N-Plex has ...

	Yes	No	
Yes	Yes	No	
No	No	Yes	

ins .C. S

If  $U_1$  ... leaves only ... rel n

- **LS-sets** (Luccio-Sami-Sets, Formalized Paradigm „Intra-cluster Coherence, inter-cluster & coherence“: An LS-set is a „network region“ level [2] internal ties are more significant than external ties (small cliques are generally closed under exclusion (Example: cycle  $C_3$ ), efficient U-sets are generally not ...)

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$u_1 \cap u_2 \neq \emptyset \rightarrow u_1 \subseteq u_2$  or  $u_2 \subseteq u_1$ ; min degree ... LS-sets is at least half number of nodes ... may be good candidates ...

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Yes	Yes	No
No	No	Yes

ins .C. S

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	Yes	No
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leave only rel n  
 If U1, U2 are adjacent, is to be adjacent to U3

LS-sets (edge disjoint), partition  
 Characteristic, inter-adjacent, network region  
 level (interactions are not significant)

sm v; U is a lambda set if  

$$\min_{u,v \in U} \lambda(u,v) > \max_{u \in U, v \in V-U} \lambda(u,v)$$

→ If an N-Plex I  
 & t U, U1, U2, interesting properties, no trivial overlap  
 $U1 \cap U2 \neq \emptyset \rightarrow U1 \subset U2 \text{ or } U2 \subset U1$ ; min degree LS is at least half  
 number of nodes

at least	No	No	Yes
ca.	No	No	Yes

