Script generated by TTT

Title: Seidl: Programmoptimierung (12.11.2015)

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Pages: 41

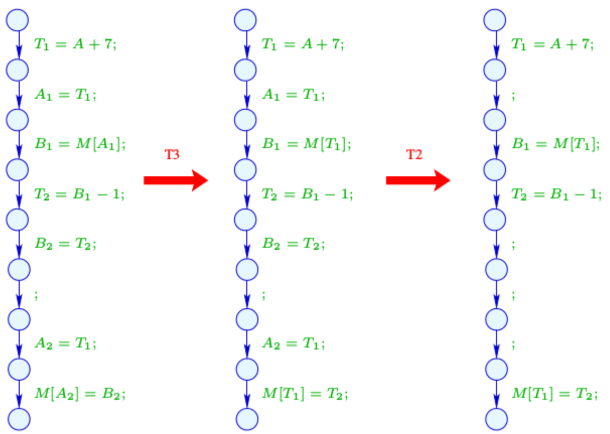
1.4 Constant Propagation

Idea:

Execute as much of the code at compile-time as possible!

Example:

Example (cont.): `a[7]--;`



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Execute as much of the code at compile-time as possible!

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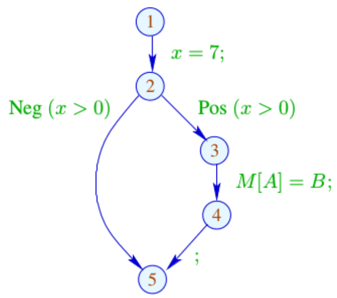
Obviously, x has always the value 7 :-)

Thus, the memory access is always executed :-))

Goal:



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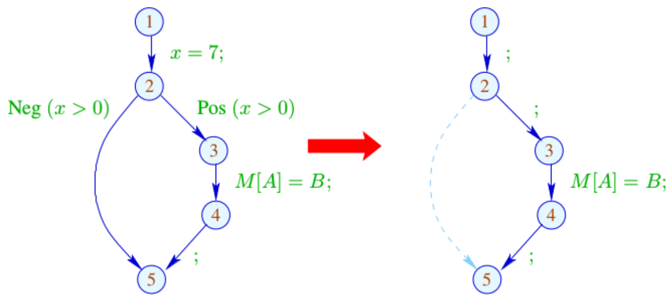

Generalization: Partial Evaluation

Neil D. Jones, DIKU, Kopenhagen

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Design an analysis which for every u,

- determines the values which variables definitely have;
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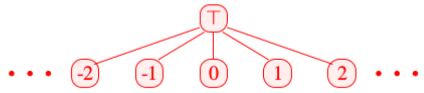
The complete lattice is constructed in two steps.

(1) The potential values of variables:

$$\mathbb{Z}^\top = \mathbb{Z} \cup \{\top\} \qquad \text{with} \quad x \sqsubseteq y \quad \text{iff } y = \top \text{ or } x = y$$



Caveat: $\mathbb{Z}^\top$ is not a complete lattice in itself :-(

(2) $\mathbb{D} = (Vars \to \mathbb{Z}^\top)_\perp = (Vars \to \mathbb{Z}^\top) \cup \{\perp\}$

// $\perp$ denotes: "not reachable" :-))

with $D_1 \sqsubseteq D_2$ iff $\perp = D_1$ or

$D_1\,x \sqsubseteq D_2\,x \quad (x \in Vars)$

Remark: $\mathbb{D}$ is a complete lattice :-)

$\{x \mapsto 5, y \mapsto \top\}$

$\perp$

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Consider $X \subseteq \mathbb{D}$. W.l.o.g., $\perp \notin X$.

Then $X \subseteq Vars \to \mathbb{Z}^\top$.

If $X = \emptyset$, then $\bigsqcup X = \perp \in \mathbb{D}$:-)

If $X \neq \emptyset$, then $\bigsqcup X = D$ with

$$D\,x = \bigsqcup\{f\,x \mid f \in X\} = \begin{cases} z & \text{if } \; f\,x = z \quad (f \in X) \\ \top & \text{otherwise} \end{cases}$$

:-))

For every edge $k = (\_, lab, \_)$, construct an effect function $[\![k]\!]^\sharp = [\![lab]\!]^\sharp \,:\, \mathbb{D} \to \mathbb{D}$ which simulates the concrete computation.

Obviously, $[\![lab]\!]^\sharp \perp = \perp$ for all lab :-)

Now let $\perp \neq D \in \mathit{Vars} \to \mathbb{Z}^\top$.

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$\{x \mapsto 5, y \mapsto \top\} \sqcup$

$\{x \mapsto 5, y \mapsto 1\} =$

$\{x \mapsto 5, y \mapsto \top\}$

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$$[\![x + a]\!]^\sharp \{x \mapsto 5, a \mapsto \top\}$$
$$= \top$$

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  $\Longrightarrow$

  We must replace the concrete operators $\Box$ by abstract operators $\Box^\sharp$ which can handle $\top$:

$$a \,\Box^\sharp\, b = \begin{cases} \top & \text{if } a = \top \text{ or } b = \top \\ a \,\Box\, b & \text{otherwise} \end{cases}$$

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- The abstract operators allow to define an abstract evaluation of expressions:

$$[\![e]\!]^\sharp \;:\; (\mathit{Vars} \to \mathbb{Z}^\top) \to \mathbb{Z}^\top$$

Abstract evaluation of expressions is like the concrete evaluation — but with abstract values and operators. Here:

$$\begin{aligned} [\![c]\!]^\sharp\, D &= c \\ [\![e_1 \,\Box\, e_2]\!]^\sharp\, D &= [\![e_1]\!]^\sharp\, D \,\Box^\sharp\, [\![e_2]\!]^\sharp\, D \end{aligned}$$

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... analogously for unary operators :-)

Example: $D = \{x \mapsto 2, y \mapsto \top\}$

$$\begin{aligned} [\![x + 7]\!]^\sharp\, D &= [\![x]\!]^\sharp\, D +^\sharp [\![7]\!]^\sharp\, D \\ &= 2 +^\sharp 7 \\ &= 9 \\ [\![x - y]\!]^\sharp\, D &= 2 -^\sharp \top \\ &= \top \end{aligned}$$

Thus, we obtain the following effects of edges $[\![lab]\!]^\sharp$:

$$\begin{aligned} [\![;]\!]^\sharp\, D &= D \\ [\![\mathrm{Pos}\,(e)]\!]^\sharp\, D &= \begin{cases} \bot & \text{if } 0 = [\![e]\!]^\sharp\, D \\ D & \text{otherwise} \end{cases} \\ [\![\mathrm{Neg}\,(e)]\!]^\sharp\, D &= \begin{cases} D & \text{if } 0 \sqsubseteq [\![e]\!]^\sharp\, D \\ \bot & \text{otherwise} \end{cases} \\ [\![x = e;]\!]^\sharp\, D &= D \oplus \{x \mapsto [\![e]\!]^\sharp\, D\} \\ [\![x = M[e];]\!]^\sharp\, D &= D \oplus \{x \mapsto \top\} \\ [\![M[e_1] = e_2;]\!]^\sharp\, D &= D \end{aligned}$$

... whenever $D \neq \bot$:-)

At *start*, we have $D_\top = \{x \mapsto \top \mid x \in \mathit{Vars}\}$.

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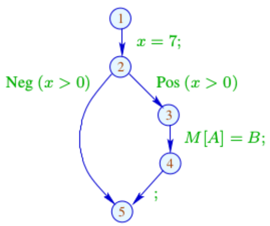

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$D = \{x \mapsto 5\}$

$[\![x > 0]\!]^\sharp D = 0$

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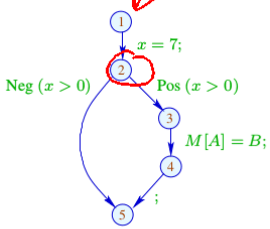

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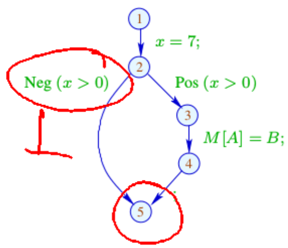

1	$\{x \mapsto \top\}$
2	$\{x \mapsto 7\}$
3	$\{x \mapsto 7\}$
4	$\{x \mapsto 7\}$
5	$\bot \sqcup \{x \mapsto 7\} = \{x \mapsto 7\}$

The abstract effects of edges $[\![k]\!]^\sharp$ are again composed to the effects of paths $\pi = k_1 \dots k_r$ by:

$$[\![\pi]\!]^\sharp = [\![k_r]\!]^\sharp \circ \dots \circ [\![k_1]\!]^\sharp \quad : \mathbb{D} \to \mathbb{D}$$

Idea for Correctness: Abstract Interpretation

Cousot, Cousot 1977

Patrick Cousot, ENS, Paris

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Idea for Correctness: Abstract Interpretation

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Establish a description relation Δ between the concrete values and their descriptions with:

$$x\,\Delta\,a_1 \quad \wedge \quad a_1 \sqsubseteq a_2 \quad \Longrightarrow \quad x\,\Delta\,a_2$$

Concretization: $\gamma\,a = \{x \mid x\,\Delta\,a\}$

// returns the set of described values :-)

(1) Values: $\Delta \subseteq \mathbb{Z} \times \mathbb{Z}^\top$

$$z\,\Delta\,a \quad \text{iff} \quad z = a \vee a = \top$$

Concretization:

$$\gamma\,a = \begin{cases} \{a\} & \text{if} \quad a \sqsubset \top \\ \mathbb{Z} & \text{if} \quad a = \top \end{cases}$$

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(2) Variable Assignments: $\Delta \subseteq (\mathit{Vars} \to \mathbb{Z}) \times (\mathit{Vars} \to \mathbb{Z}^\top)_\perp$

$$\rho\,\Delta\,D \quad \text{iff} \quad D \neq \perp \wedge \rho\,x\ \Delta\ D\,x \quad (x \in \mathit{Vars})$$

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Example: $\{x \mapsto 1, y \mapsto -7\} \;\Delta\; \{x \mapsto \top, y \mapsto -7\}$

(3) States:

$$\Delta \subseteq ((\mathit{Vars} \to \mathbb{Z}) \times (\mathbb{N} \to \mathbb{Z})) \times (\mathit{Vars} \to \mathbb{Z}^\top)_\perp$$

$$(\rho, \mu) \,\Delta\, D \quad \text{iff} \quad \rho \,\Delta\, D$$

Concretization:

$$\gamma\, D = \begin{cases} \emptyset & \text{if} \quad D = \perp \\ \{(\rho, \mu) \mid \forall\, x :\; (\rho\, x) \,\Delta\, (D\, x)\} & \text{otherwise} \end{cases}$$

We show:

$(*)$ If $s \,\Delta\, D$ and $[\![\pi]\!]\, s$ is defined, then:

$$([\![\pi]\!]\, s) \;\Delta\; ([\![\pi]\!]^\sharp\, D)$$

$$\begin{array}{ccc} s & \xrightarrow{[\![\pi]\!]} & s_1 \\ \Delta & & \Delta \\ D & \xrightarrow[{[\![\pi]\!]^\sharp}]{} & D_1 \end{array}$$

The abstract semantics simulates the concrete semantics :-)

In particular:

$$[\![\pi]\!]\, s \in \gamma\, ([\![\pi]\!]^\sharp\, D)$$

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In practice, this means, e.g., that $D\, x = -7$ implies:

$$\begin{array}{llll} & \rho'\, x & = -7 & \text{for all} \quad \rho' \in \gamma\, D \\ \Longrightarrow & \rho_1\, x & = -7 & \text{for} \quad (\rho_1, \_) = [\![\pi]\!]\, s \end{array}$$

To prove $(*)$, we show for every edge k :

$[\![2]\!]\; s \;\Delta\; [\![2]\!]^\sharp\, D$

2

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$$(**) \qquad \begin{tikzcd} s \arrow[r, "{[\![k]\!]}"] \arrow[d, dash, "\Delta"'] & s_1 \arrow[d, dash, dotted, "\Delta"] \\ D \arrow[r, "{[\![k]\!]^\sharp}"'] & D_1 \end{tikzcd}$$

Then $(*)$ follows by induction :-)

To prove $(**)$, we show for every expression e :

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This precisely was how we have defined the operators $\Box^\sharp$:-)