Script **generated by TTT**

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Lists

From the Prelude:

```
data [a] = [] | (:) a [a]
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The result of deriving Eq:

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instance Eq a => Eq [a] where
  []     == []      =  True
  (x:xs) == (y:ys)  =  x == y && xs == ys
  _      == _       =  False
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Defined explicitly:


```
instance Show a => Show [a] where
  show xs  =  "[" ++ concat cs ++ "]"
    where cs = Data.List.intersperse ", "  (map show xs)
```

Tree

```
data Tree a = Empty | Node a (Tree a) (Tree a)
              deriving (Eq, Show)
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Some trees:

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Empty
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Some trees:

```
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Node 1 Empty Empty
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Some trees:

```
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Node 1 Empty Empty
Node 1 (Node 2 Empty Empty) Empty
Node 1 Empty (Node 2 Empty Empty)
Node 1 (Node 2 Empty Empty) (Node 3 Empty Empty)
```

Tree

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data Tree a = Empty | Node a (Tree a) (Tree a)
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Some trees:

```
Empty
Node 1 Empty Empty
Node 1 (Node 2 Empty Empty) Empty
Node 1 Empty (Node 2 Empty Empty)
Node 1 (Node 2 Empty Empty) (Node 3 Empty Empty)
⋮
```

```
find ::          a -> Tree a -> Bool
```

```
find :: Ord a => a -> Tree a -> Bool
find _ Empty  =  False
```

```
find :: Ord a => a -> Tree a -> Bool
find _ Empty  =  False
find x (Node a l r)
```

```
find :: Ord a => a -> Tree a -> Bool
find _ Empty  =  False
find x (Node a l r)
  | x < a   =  find x l
  | a < x   =  find x r
  | otherwise  =
```

```
find :: Ord a => a -> Tree a -> Bool
find _ Empty  =  False
find x (Node a l r)
  | x < a  =  find x l
  | a < x  =  find x r
  | otherwise  =  True
```

```
-- assumption: < is a linear ordering
find :: Ord a => a -> Tree a -> Bool
find _ Empty  =  False
find x (Node a l r)
  | x < a  =  find x l
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```

```
insert :: Ord a => a -> Tree a -> Tree a
```

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insert :: Ord a => a -> Tree a -> Tree a
insert x Empty  =  Node x Empty Empty
insert x (Node a l r)
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Example

```haskell
insert 6 (Node 5 Empty (Node 7 Empty Empty))
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```

Example

```haskell
insert 6 (Node 5 Empty (Node 7 Empty Empty))
= Node 5 Empty (insert 6 (Node 7 Empty Empty))
= Node 5 Empty (Node 7 (insert 6 Empty) Empty)
```

QuickCheck for Tree

```haskell
import Control.Monad
import Test.QuickCheck

-- for QuickCheck: test data generator for Trees
instance Arbitrary a => Arbitrary (Tree a) where
  arbitrary = sized tree
    where
    tree 0 = return Empty
    tree n | n > 0 =
      oneof [return Empty,
             liftM3 Node arbitrary (tree (n `div` 2))
                                   (tree (n `div` 2))]
```

```
insert :: Ord a => a -> Tree a -> Tree a
insert x Empty  =  Node x Empty Empty
insert x (Node a l r)
  | x < a  =  Node a (insert x l) r
  | a < x  =  Node a l (insert x r)
  | otherwise  =  Node a l r
```

Example

```
insert 6 (Node 5 Empty (Node 7 Empty Empty))
= Node 5 Empty (insert 6 (Node 7 Empty Empty))
= Node 5 Empty (Node 7 (insert 6 Empty) Empty)
= Node 5 Empty (Node 7 (Node 6 Empty Empty) Empty)
```

QuickCheck for Tree

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import Control.Monad
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                                   (tree (n `div` 2))]
```

```
prop_find_insert x y t =
  find x (insert y t) == ???
```

```
prop_find_insert x y t =
  find x (insert y t) == (x == y || find x t)
```

```
prop_find_insert ::
prop_find_insert x y t =
  find x (insert y t) == (x == y || find x t)
```

```
prop_find_insert :: Int -> Int -> Tree Int -> Bool
prop_find_insert x y t =
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prop_find_insert :: Int -> Int -> Tree Int -> Bool
prop_find_insert x y t =
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```

(`Int` not optimal for QuickCheck)

Edit distance (see Thompson)

Problem: how to get from one word to another,
with a *minimal* number of “edits”.

Example: from `"fish"` to `"chips"`

Applications: DNA Analysis,

Edit distance (see Thompson)

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with a *minimal* number of "edits".

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Applications: DNA Analysis, Unix `diff` command

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data Edit = Change Char
          | Copy
          | Delete
          | Insert Char
          deriving (Eq, Show)
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transform :: String -> String -> [Edit]
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transform [] ys  =
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transform :: String -> String -> [Edit]

transform [] ys  =  map Insert ys
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data Edit = Change Char
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          | Delete
          | Insert Char
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transform :: String -> String -> [Edit]

transform [] ys  =  map Insert ys
transform xs []  =  replicate (length xs) Delete
transform (x:xs) (y:ys)
  | x == y       =  Copy
```

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data Edit = Change Char
          | Copy
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          | Insert Char
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transform :: String -> String -> [Edit]

transform [] ys  =  map Insert ys
transform xs []  =  replicate (length xs) Delete
transform (x:xs) (y:ys)
  | x == y       =  Copy : transform xs ys
```

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transform :: String -> String -> [Edit]

transform [] ys  =  map Insert ys
transform xs []  =  replicate (length xs) Delete
transform (x:xs) (y:ys)
  | x == y       =  Copy : transform xs ys
  | otherwise    =  best
```

```
best :: [[Edit]] -> [Edit]
best [x]    = x
best (x:xs)
```

```
best :: [[Edit]] -> [Edit]
best [x]    = x
best (x:xs)
  | cost x <= cost b     = x
  | otherwise            = b
  where b = best xs

cost :: [Edit] -> Int
cost = length . filter (/=Copy)
```

Example: What is the edit distance from `"trittin"` to `"tarantino"`?

```
best :: [[Edit]] -> [Edit]
best [x]    = x
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  | otherwise    =  best [Change y : transform xs ys,
                          Delete   : transform xs (y:ys),
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Complexity of `transform`: time $O($

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The edit distance problem can be solved in time $O(mn)$
with *dynamic programming*

8.2 The general case

data T a_1 ... a_p =
  C_1 t_{11} ... t_{1k_1} |
  ⋮
  C_n t_{n1} ... t_{nk_n}

defines the *constructors*

C_1 :: t_{11} -> ... t_{1k_1} -> T a_1 ... a_p
⋮
C_n :: t_{n1} -> ... t_{nk_n} -> T a_1 ... a_p

Constructors are functions too!

Constructors can be used just like other functions

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  | x == y        =  Copy : transform xs ys
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                           Delete   : transform xs (y:ys),
                           Insert y : transform (x:xs) ys]
```

Constructors are functions too!

Constructors can be used just like other functions

Example

```
map Just [1, 2, 3]  =  [Just 1, Just 2, Just 3]
```

But constructors can *also* occur in patterns!

Patterns revisited

Patterns are expressions that consist only of constructors and variables (which must not occur twice):
A *pattern* can be

- a variable (incl. `_`)
- a literal like `1`, `'a'`, `"xyz"`, . . .

Patterns revisited

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- a literal like `1`, `'a'`, `"xyz"`, . . .

- a tuple `(`p_1`, ..., `p_n`)` where each p_i is a pattern

.3 Case study: boolean formulas

Patterns revisited

Patterns are expressions that consist only of constructors and variables (which must not occur twice):
A *pattern* can be

- a variable (incl. `_`)
- a literal like `1`, `'a'`, `"xyz"`, . . .
- a tuple `(`p_1`, ..., `p_n`)` where each p_i is a pattern

- a constructor pattern C p_1 ... p_n where
  C is a `data` constructor (incl. `True`, `False`, `[]` and `(:)`)
  and each p_i is a pattern

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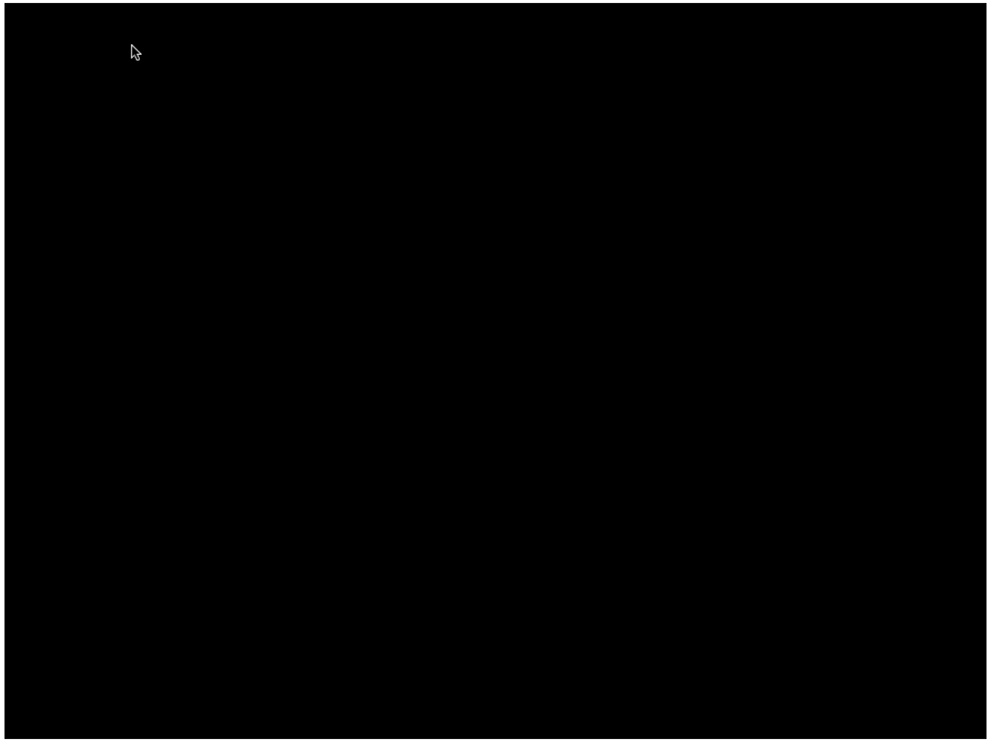

TTT Fri 09:41


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