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Inefficiency of `reverse`

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reverse (x:xs) = reverse xs ++ [x]
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= reverse [2,3] ++ [1]
= (reverse [3] ++ [2]) ++ [1]
= ((reverse [] ++ [3]) ++ [2]) ++ [1]
= (([] ++ [3]) ++ [2]) ++ [1]
= ([3] ++ [2]) ++ [1]
= (3 : ([] ++ [2])) ++ [1]
= [3,2] ++ [1]
= 3 : ([2] ++ [1])
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= [3,2,1]
```

An improvement: `itrev`

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Lemma `itrev xs [] = reverse xs`

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itrev (x:xs) []
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Problem: IH not applicable because too specialized: `[]`

Generalization

Lemma `itrev xs ys =`

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Justification: all variables are implicitly ∀-quantified, except for the induction variable.

Induction on the length of a list

```
qsort :: Ord a => [a] -> [a]
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qsort :: Ord a => [a] -> [a]
qsort []      =  []
qsort (x:xs)  = qsort below ++ [x] ++ qsort above
    where below = [y | y <- xs, y <= x]
          above = [z | y <- xs, x < z]
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```

Lemma `qsort xs` is sorted

Proof by induction on the length of the argument of `qsort`.

Is that all? Or should we prove something else about sorting?

How about this sorting function?

`superquicksort _ = []`

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`superquicksort _ = []`

Every element should occur as often in the output as in the input!

5.2 Definedness

Simplifying assumption, implicit so far:

No undefined values

Two kinds of undefinedness:

`head []` raises exception

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Undefinedness can be handled, too.

But it complicates life

What is the problem?

Many familiar laws no longer hold unconditionally:

`x - x = 0`

is true only if `x` is a defined value.

Two examples:

- Not true: `head [] - head [] = 0`
- From the nonterminating definition
  `f x = f x + 1`
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Termination

Termination of a function means termination for all inputs.

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Restriction:

The proof methods in this chapter assume that all recursive definitions under consideration terminate.

Most Haskell functions we have seen so far terminate.

How to prove termination

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if there is a *measure function* `m :: T1 ->` $\mathbb{N}$ such that

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Note:

- All primitive recursive functions terminate.

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- for every defining equation `f p = t`
- and for every recursive call `f r` in `t`: `m p > m r`.

Note:

- All primitive recursive functions terminate.

- `m` can be defined in Haskell or mathematics.

More generally: `f :: T1 -> ... -> Tn -> T` terminates
if there is a measure function `m :: T1 -> ... -> Tn -> ℕ`
such that

- for every defining equation `f p1 ... pn = t`
- and for every recursive call `f r1 ... rn` in `t`:
  `m p1 ... pn > m r1 ... rn`.

Of course, all other functions that are called by `f` must also terminate.

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Example: `[1, 1, 1, ...]`

Infinite objects must be constructed by recursion:

```
ones = 1 : ones
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Because we restrict to terminating definitions in this chapter, infinite values cannot arise.

Note:

- By termination of functions we really mean termination on *finite* values.
- For example `reverse` terminates only on finite lists.

How can infinite values be useful?
Because of “lazy evaluation”.

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More later.

Exceptions

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In such cases, we can prove equations `e1 = e2` that are only *partially correct*:

Summary

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- In this chapter everything must terminate
- This avoids undefined and infinite values
- This simplifies proofs

5.3 Interlude: Type inference/reconstruction

How to infer/reconstruct the type of an expression
(and all subexpressions)

Recall [Pic is short for Picture]

```
alterH :: Pic -> Pic -> Int -> Pic
alterH pic1 pic2 1 = pic1
alterH pic1 pic2 n = beside pic1 (alterH pic2 pic1 (n-1))

alterV :: Pic -> Pic -> Int -> Pic
alterV pic1 pic2 1 = pic1
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Very similar. Can we avoid duplication?

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Higher-order functions capture patterns of computation

6.1 Applying functions to all elements of a list: `map`

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map even [1, 2, 3]
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What is the type of `map`?

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map ::          ->      ->
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Two possible definitions:

```
map f xs   =  [ f x | x <- xs ]
```

Evaluating map

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map f []       =  []
map f (x:xs)   =  f x : map f xs

map sqr [1, -2]
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map sqr [1, -2]
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= sqr 1 : map sqr (-2 : [])
```

Evaluating map

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map f []      =  []
map f (x:xs)  =  f x : map f xs

map sqr [1, -2]
= map sqr (1 : -2 : [])
= sqr 1 : map sqr (-2 : [])
= sqr 1 : sqr (-2) : (map sqr [])
= sqr 1 : sqr (-2) : []
= 1 : 4 : []
= [1, 4]
```

Some properties of map

```
length (map f xs) = length xs
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Proofs by induction

QuickCheck and function variables

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It needs to know how to generate and print functions.

Cheap alternative: replace function variable by specific function(s)

Example

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prop_map_even :: [Int] -> [Int] -> Bool
prop_map_even xs ys =
  map even (xs ++ ys) = map even xs ++ map even ys
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filter null [[], [1,2], []]
= [[], []]
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What is the type of `filter`?

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filter :: (a -> Bool) -> [a] ->
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`filter`

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Two possible definitions:

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filter p xs  =  [ x | x <- xs, p x ]
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filter p (map f xs) = map f (filter p xs)
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Given: an expression *e*

Type inference:

1. Give all variables and functions in *e* their most general type
2. From *e* set up a system of equations between types
3. Simplify the equations

Example: `concat (replicate x y)`

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Initial type table:

```
x ::  a
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x ::  a
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replicate ::  Int -> c -> [c]
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Simplify equations: `[c] = [[d]]` ⇝

`c = [d]`

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`b = c` ⇝ `b = [d]`

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Solution to equation system: `a = Int, b = [d], c = [d]`

Final type table:

```
x :: Int
y :: [d]
replicate :: Int -> [d] -> [[d]]
```

Example: `concat (replicate x y)`

Initial type table:

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x :: a
y :: b
replicate :: Int -> c -> [c]
concat :: [[d]] -> [d]
```

For each subexpression $f\ e_1 \ldots e_n$ generate n equations:

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a = Int, b = c
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   - $a = T \ldots a \ldots$ or $T \ldots a \ldots = a$: type error!
   - $T \ldots = T' \ldots$ where $T \neq T'$: type error!

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- For simple expressions you should be able to infer types "durch scharfes Hinsehen"
- Use the algorithm if you are unsure or the expression is complicated
- Or use the Haskell interpreter

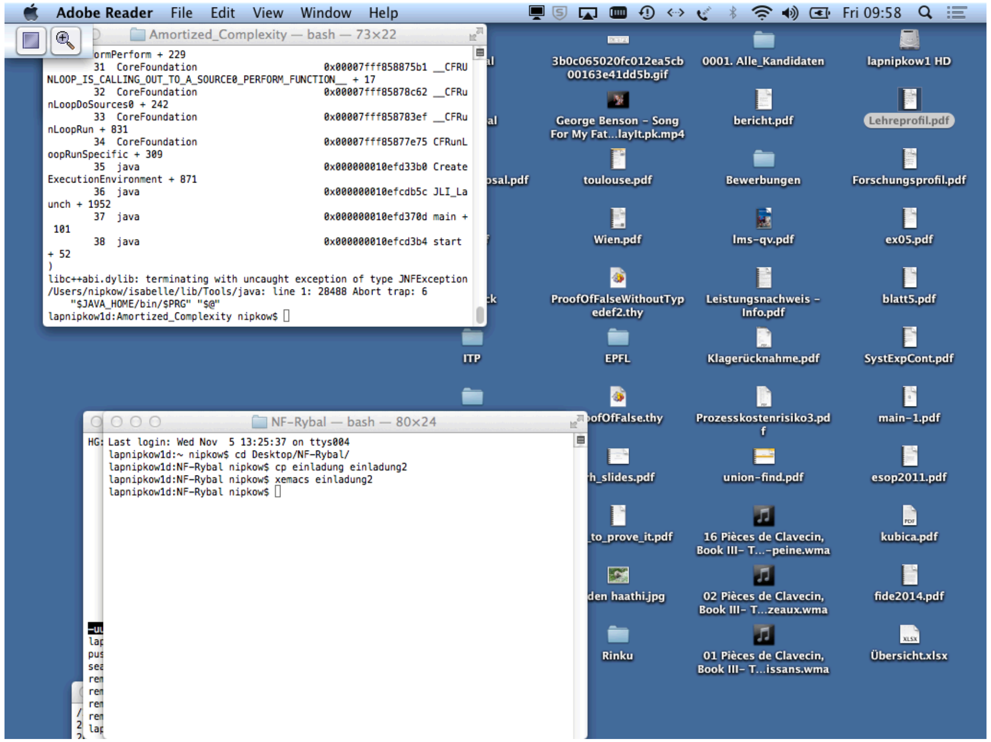
Some properties of `filter`

True or false?

```
filter p (xs ++ ys) = filter p xs ++ filter p ys

filter p (reverse xs) = reverse (filter p xs)

filter p (map f xs) = map f (filter p xs)
```


Adobe Reader File Edit View Window Help
Fri 09:58
Amortized_Complexity — bash — 73×22
ormPerform + 229
31 CoreFoundation 0x00007fff858875b1 __CFRU
NLOOP_IS_CALLING_OUT_TO_A_SOURCE0_PERFORM_FUNCTION__ + 17
32 CoreFoundation 0x00007fff85878c62 __CFRu
nLoopDoSources0 + 242
33 CoreFoundation 0x00007fff858783ef __CFRu
nLoopRun + 831
34 CoreFoundation 0x00007fff85877e75 CFRunL
oopRunSpecific + 309
35 java 0x000000010efd33b0 Create
ExecutionEnvironment + 871
36 java 0x000000010efcdb5c JLI_La
unch + 1952
37 java 0x000000010efd370d main +
101
38 java 0x000000010efcd3b4 start
+ 52
)
libc++abi.dylib: terminating with uncaught exception of type JNFException
/Users/nipkow/isabelle/lib/Tools/java: line 1: 28488 Abort trap: 6
"$JAVA_HOME/bin/$PRG" "$@"
lapnipkow1d:Amortized_Complexity nipkow$
NF-Rybal — bash — 80×24
Last login: Wed Nov 5 13:25:37 on ttys004
lapnipkow1d:~ nipkow$ cd Desktop/NF-Rybal/
lapnipkow1d:NF-Rybal nipkow$ cp einladung einladung2
lapnipkow1d:NF-Rybal nipkow$ xemacs einladung2
lapnipkow1d:NF-Rybal nipkow$
3b0c065020fc012ea5cb00163e41dd5b.gif
0001. Alle_Kandidaten
lapnipkow1 HD
George Benson - Song For My Fat...laylt.pk.mp4
bericht.pdf
Lehreprofil.pdf
toulouse.pdf
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Forschungsprofil.pdf
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