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TTT Tue 15:30 Tobias Nipkow

.3 Case study: boolean formulas

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type Name = String
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type Name = String

data Form = F | T
          | Var Name
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          | And Form Form
          | Or Form Form
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.3 Case study: boolean formulas

```
type Name = String

data Form = F | T
          | Var Name
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          deriving Eq
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Example: `Or (Var "p") (Not(Var "p"))`

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More readable: symbolic infix constructors, must start with :

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data Form = F | T | Var Name
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Now: `Var "p" :|: Not(Var "p")`

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par s = "(" ++ s ++ ")"
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  show F = "F"
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  show (p :&: q) = par(show p ++ " & " ++ show q)
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Syntax versus meaning

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`Form` is the *syntax* of boolean formulas, not their meaning:

`Not(Not T)` and `T` mean the same but are different:

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What is the meaning of a `Form`?

Its value!?

But what is the value of `Var "p"` ?

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-- Wertebelegung
type Valuation = [(Name,Bool)]
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eval :: Valuation -> Form -> Bool
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> eval [("a",False), ("b",False)]
   (Not(Var "a") :&: Not(Var "b"))
```

All valuations for a given list of variable names:

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vals :: [Name] -> [Valuation]
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              [ (x,True):v  | v <- vals xs ]
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vals ["a","b"]
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= [[("a",False),("b",False)] ++ [("a",False),("b",True)] ++
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prop_vals2 xs =
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distinct :: Eq a => [a] -> Bool
distinct [] = True
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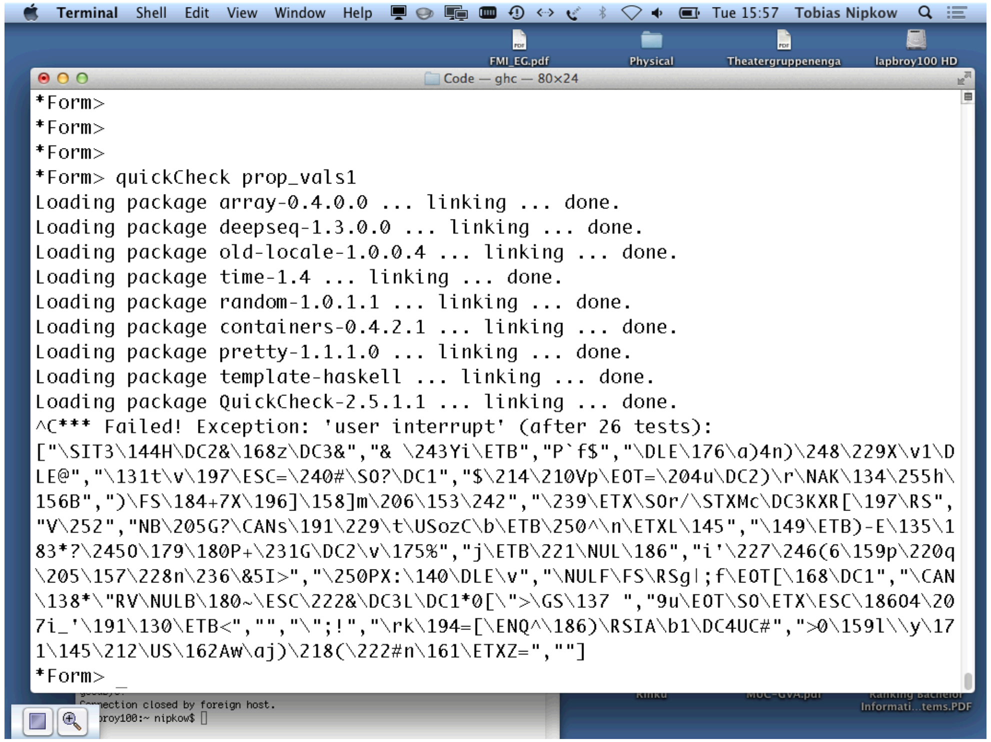
Demo



Restrict size of test cases:

```
prop_vals1' xs =
  length xs <= 10 ==>
  length(vals xs) == 2 ^ length xs

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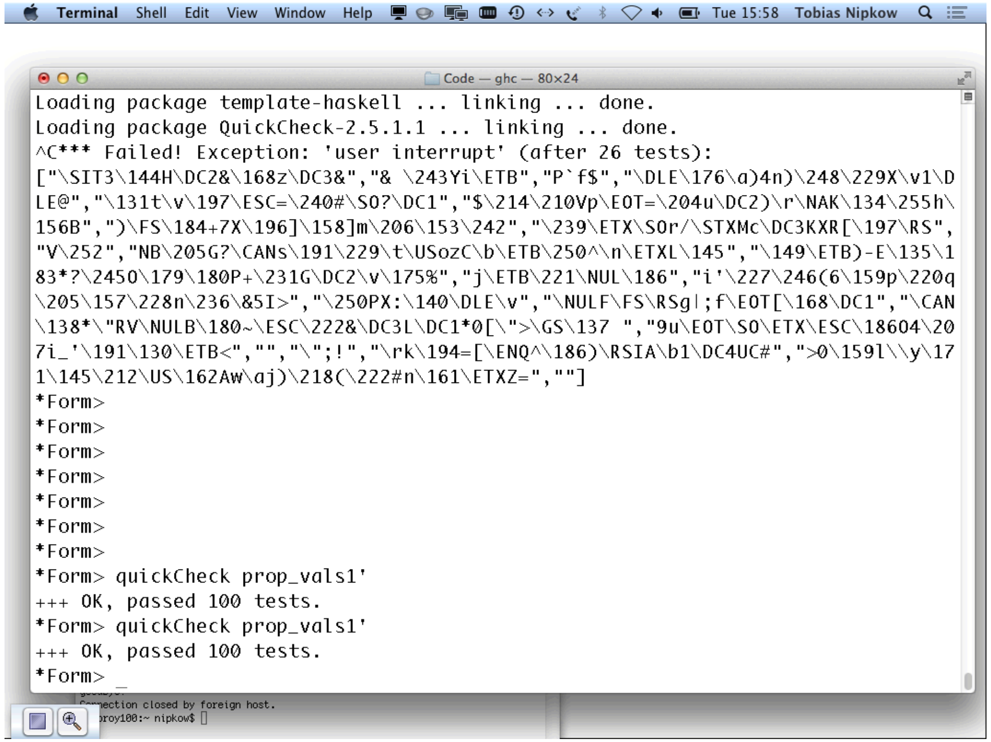

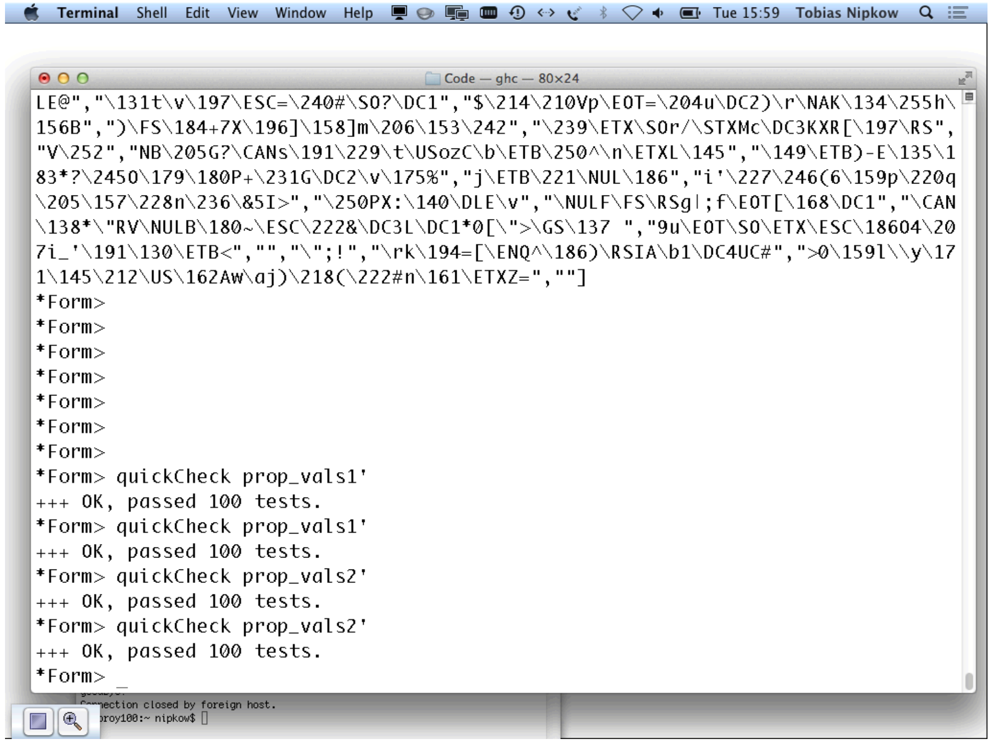

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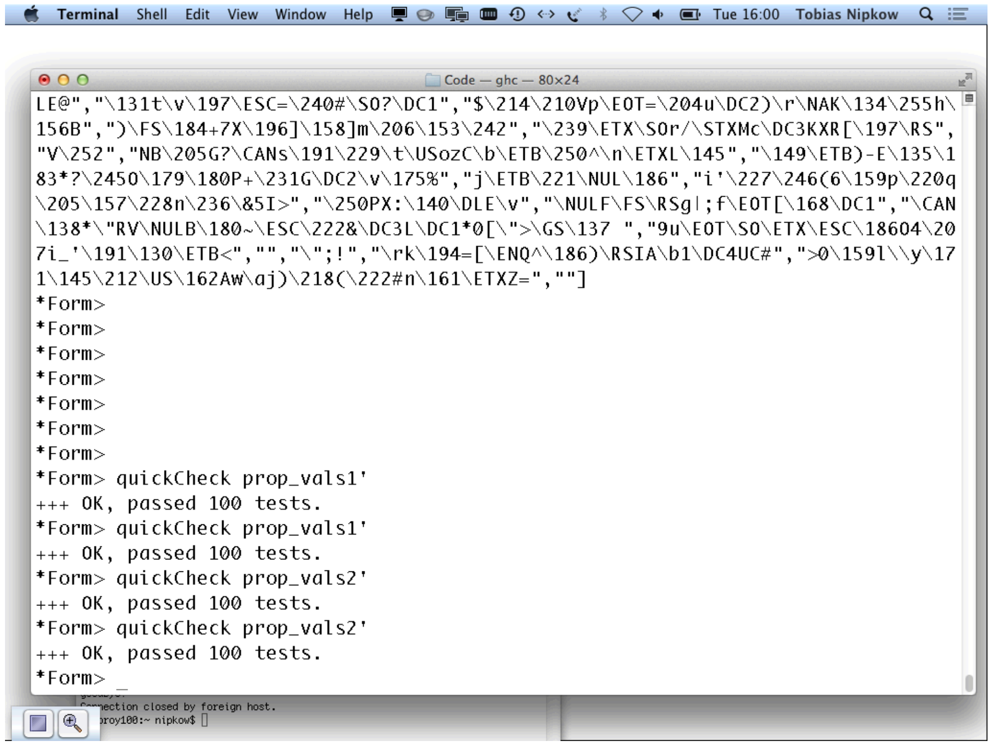

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Satisfiable and tautology

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```
vars :: Form -> [Name]
vars F = []
vars T = []
vars (Var x) = [x]
vars (Not p) = vars p
vars (p :&: q) = nub (vars p ++ vars q)
vars (p :|: q) = nub (vars p ++ vars q)
```

```
p0 :: Form
p0 =  (Var "a" :&: Var "b") :|:
      (Not (Var "a") :&: Not (Var "b"))
```

Simplifying a formula: Not inside?

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isSimple :: Form -> Bool
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  isOp p           =  False
isSimple (p :&: q)  =  isSimple p && isSimple q
isSimple (p :|: q)  =  isSimple p && isSimple q
isSimple p          =  True
```

Simplifying a formula: Not inside!

```
simplify :: Form -> Form
simplify (Not p)     =  pushNot (simplify p)
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  pushNot (p :|: q)  =  pushNot p :&: pushNot q
  pushNot p          =  Not p
simplify (p :&: q)  =  simplify q :&: simplify q
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Simplifying a formula: Not inside!

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simplify (Not p)    =  pushNot (simplify p)
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simplify (p :&: q)  =  simplify q :&: simplify q
simplify (p :|: q)  =  simplify p :|: simplify q
simplify p          =
```

Quickcheck

```
-- for QuickCheck: test data generator for Form
instance Arbitrary Form where
  arbitrary = sized prop
    where
    prop 0  =
      oneof [return F,
             return T,
             liftM Var arbitrary]
    prop n | n > 0 =
      oneof
        [return F,
         return T,
         liftM Var arbitrary,
         liftM Not (prop (n-1)),
         liftM2 (:&:) (prop(n `div` 2)) (prop(n `div` 2)),
         liftM2 (:|:) (prop(n `div` 2)) (prop(n `div` 2))]
```

```
prop_simplify p  =  isSimple(simplify p)
```

Simplifying a formula: Not inside!

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simplify :: Form -> Form
simplify (Not p)     =  pushNot (simplify p)
  where
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  pushNot (p :|: q)   =  pushNot p :&: pushNot q
  pushNot p           =  Not p
simplify (p :&: q)   =  simplify q :&: simplify q
simplify (p :|: q)   =  simplify p :|: simplify q
simplify p           =  p
```

8.4 Structural induction

Structural induction for Tree

```
data Tree a = Empty | Node a (Tree a) (Tree a)
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To prove property `P(t)` for all finite `t :: Tree a`

Base case: Prove `P(Empty)` and

Induction step: Prove `P(Node x t1 t2)`
assuming the induction hypotheses `P(t1)` and `P(t2)`.

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Base case: Prove `P(Empty)` and

Induction step: Prove `P(Node x t1 t2)`
assuming the induction hypotheses `P(t1)` and `P(t2)`.
(`x`, `t1` and `t2` are new variables)

Example

```
flat :: Tree a -> [a]
flat Empty  =  []
flat (Node x t1 t2)  =
  flat t1 ++ [x] ++ flat t2

mapTree :: (a -> b) -> Tree a -> Tree b
mapTree f Empty  =  Empty
mapTree f (Node x t1 t2)  =
  Node (f x) (mapTree f t1) (mapTree f t2)
```

Lemma `flat (mapTree f t) = map f (flat t)`

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Proof by structural induction on `t`

Induction step:

IH1: `flat (mapTree f t1) = map f (flat t1)`
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Lemma `flat (mapTree f t) = map f (flat t)`

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To show: `flat (mapTree f (Node x t1 t2)) =`
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Lemma `flat (mapTree f t) = map f (flat t)`

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```
flat (mapTree f (Node x t1 t2))
= flat (Node (f x) (mapTree f t1) (mapTree f t2))
```

Lemma `flat (mapTree f t) = map f (flat t)`

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To show: `flat (mapTree f (Node x t1 t2)) =`
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```
flat (mapTree f (Node x t1 t2))
= flat (Node (f x) (mapTree f t1) (mapTree f t2))
```


```
= flat (mapTree f t1) ++ [f x] ++ flat (mapTree f t2)
```

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Proof by structural induction on `t`

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flat (mapTree f (Node x t1 t2))
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To show: `flat (mapTree f (Node x t1 t2)) =`
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```
flat (mapTree f (Node x t1 t2))
= flat (Node (f x) (mapTree f t1) (mapTree f t2))
= flat (mapTree f t1) ++ [f x] ++ flat (mapTree f t2)
= map f (flat t1) ++ [f x] ++ map f (flat t2)
    -- by IH1 and IH2
map f (flat (Node x t1 t2))
= map f (flat t1 ++ [x] ++ flat t2)
```

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IH1: `flat (mapTree f t1) = map f (flat t1)`
IH2: `flat (mapTree f t2) = map f (flat t2)`

To show: `flat (mapTree f (Node x t1 t2)) =`
`map f (flat (Node x t1 t2))`

```
flat (mapTree f (Node x t1 t2))
= flat (Node (f x) (mapTree f t1) (mapTree f t2))
= flat (mapTree f t1) ++ [f x] ++ flat (mapTree f t2)
= map f (flat t1) ++ [f x] ++ map f (flat t2)
    -- by IH1 and IH2
map f (flat (Node x t1 t2))
= map f (flat t1 ++ [x] ++ flat t2)
= map f (flat t1) ++ [f x] ++ map f (flat t2)
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data T a = ...
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Structural induction for `Tree`

```
data Tree a = Empty | Node a (Tree a) (Tree a)
```

To prove property `P(t)` for all finite `t :: Tree a`

Base case: Prove `P(Empty)` and

Induction step: Prove `P(Node x t1 t2)`
assuming the induction hypotheses `P(t1)` and `P(t2)`.

- So far, only batch programs:
  given the full input at the beginning,
  the full output is produced at the end
- Now, interactive programs:
  read input and write output
  while the program is running

The problem

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  Haskell programs have no side effects

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```
inputInt - inputInt   =   0
```

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`IO Char`: the type of actions that return a `Char`

`IO ()`: the type of actions that return no result value

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- Therefore `IO ()` is the type of actions that return the dummy value `()`

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  Reads a `Char` from standard input, echoes it to standard output, and returns it as the result
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  Writes a `Char` to standard output,
  and returns no result
- `return :: a -> IO a`

  Performs no action,
  just returns the given value as a result

Sequencing: do

A sequence of actions can be combined into a single action with the keyword do

Example

```
get2 :: IO  ?
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get2 = do x <- getChar    -- result is named x
          getChar
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Example

```
get2 :: IO  ?
get2 = do x <- getChar    -- result is named x
          getChar         -- result is ignored
          y <- getChar
          return (x,y)
```

General format (observe layout!):

do a_1
$\vdots$
a_n

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- `let` x `=` *expr*
  Effect: give *expr* the name x
  Lazy: *expr* is only evaluated when x is needed!

Derived primitives

Write a string to standard output:

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putStr :: String -> IO ()
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putStrLn cs   =  putStr (cs ++ '\n')
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getLine = do x <- getChar
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             else
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```

Actions are normal Haskell values and can be combined as usual, for example with `if-then-else`.

Derived primitives

Write a string to standard output:

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putStr :: String -> IO ()
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Example

Prompt for a string and display its length:

```
strLen :: IO ()
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               xs <- getLine
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How to read other types

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Input string and convert

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Useful class:

```
class Read a where
  read :: String -> a
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```
class Read a where
  read :: String -> a
```

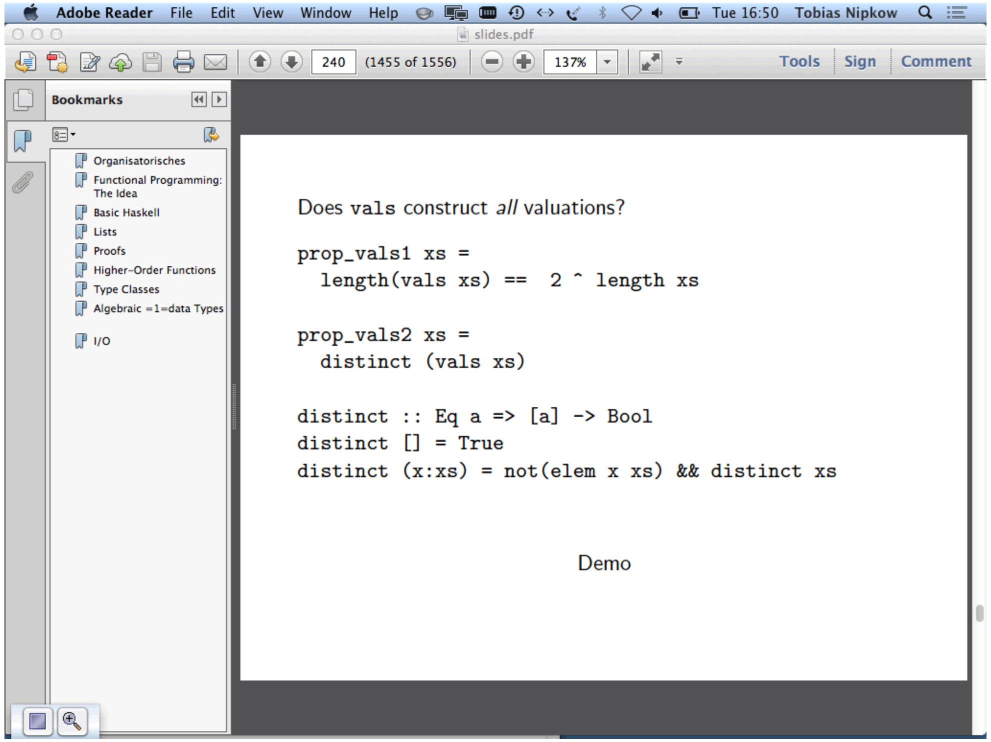
Most predefined types are in class Read.

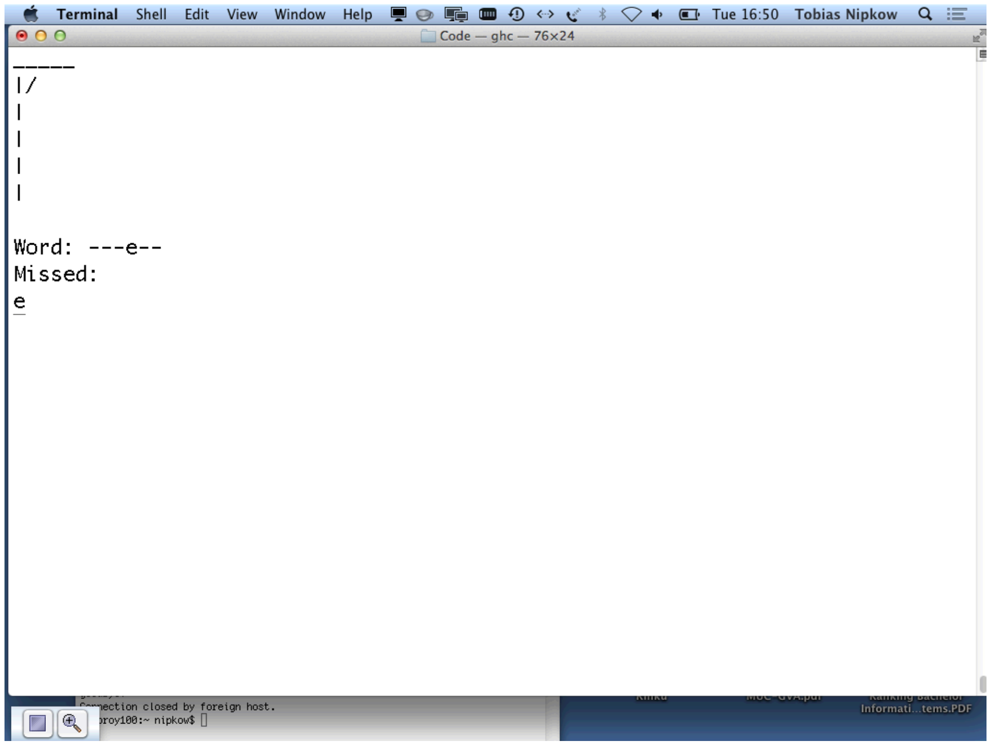
Example:

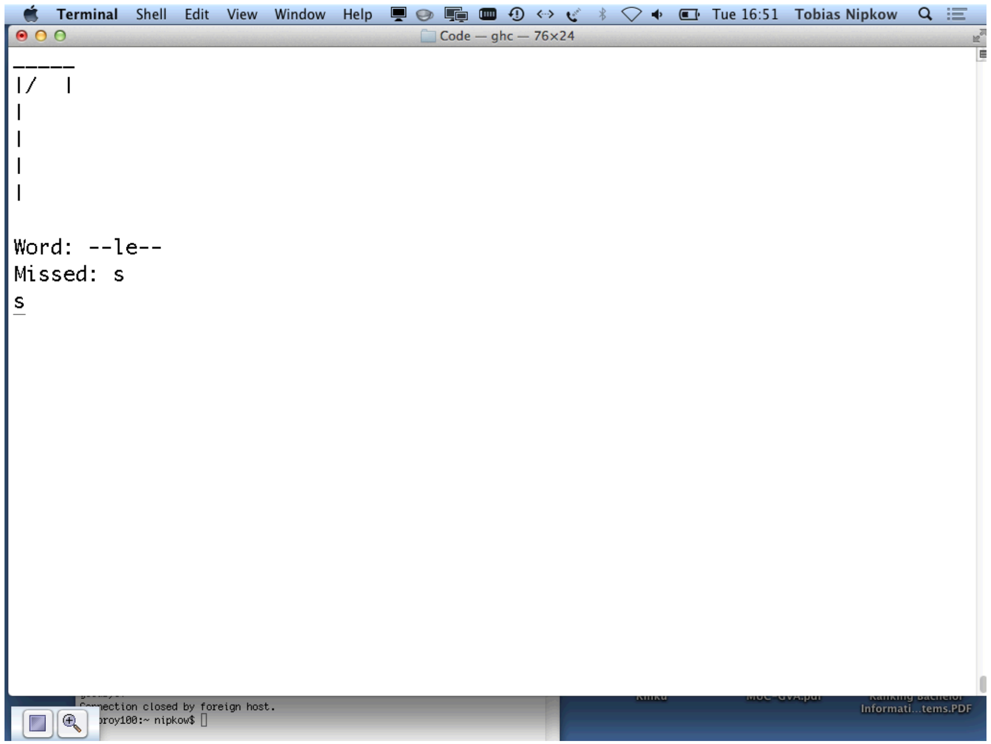
```
getInt :: IO Integer
getInt = do xs <- getLine
            return (read xs)
```

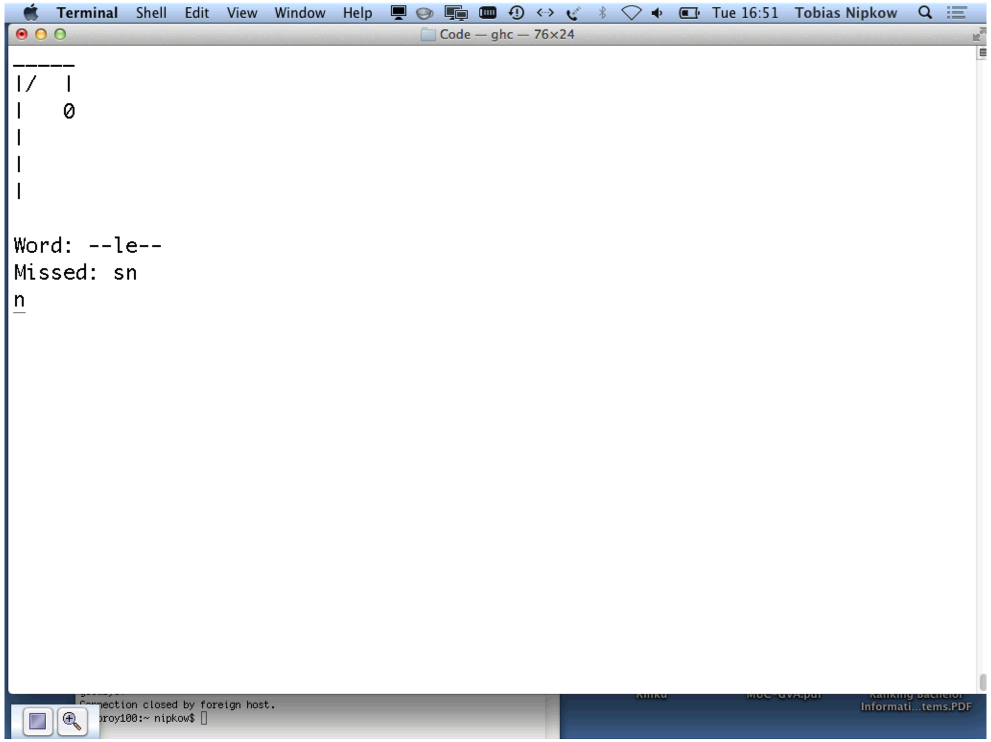
Case study

The game of Hangman
in file `hangman.hs`

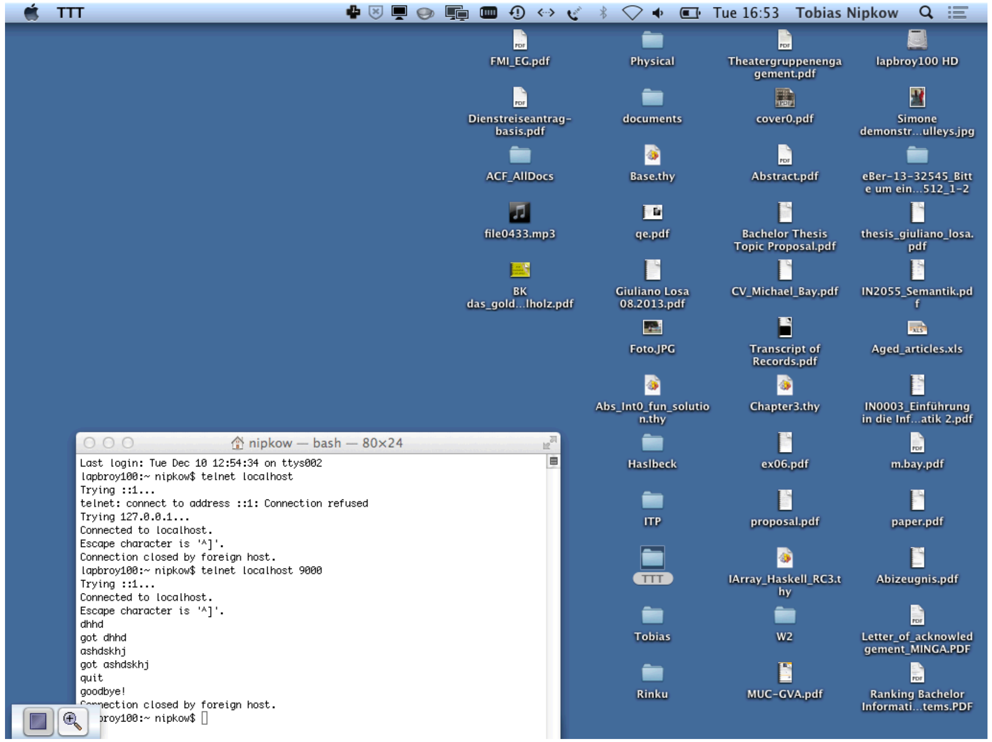








Terminal Shell Edit View Window Help Tue 16:51 Tobias Nipkow
Code — ghc — 76×24
_____
|/ |
| 0
|
|
|
Word: --le--
Missed: sn
n
Connection closed by foreign host.
lapbroy100:~ nipkow$



TTT Tue 16:53 Tobias Nipkow
nipkow — bash — 80×24
Last login: Tue Dec 10 12:54:34 on ttys002
lapbroy100:~ nipkow$ telnet localhost
Trying ::1...
telnet: connect to address ::1: Connection refused
Trying 127.0.0.1...
Connected to localhost.
Escape character is '^]'.
Connection closed by foreign host.
lapbroy100:~ nipkow$ telnet localhost 9000
Trying ::1...
Connected to localhost.
Escape character is '^]'.
dhhd
got dhhd
ashdskhj
got ashdskhj
quit
goodbye!
Connection closed by foreign host.
lapbroy100:~ nipkow$