

**Script** generated by TTT

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# Functional Data Structures

with Isabelle/HOL

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## Part II

# Functional Data Structures

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- ① Correctness
- ② Insertion Sort
- ③ Time
- ④ Merge Sort

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$sorted :: ('a::linorder) list \Rightarrow bool$

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$sorted [] = True$

$sorted (x \# ys) = ((\forall y \in set\ ys. x \leq y) \wedge sorted\ ys)$

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## Correctness of sorting

Specification of  $sort :: ('a::linorder) list \Rightarrow 'a\ list$ :

$sorted (sort\ xs)$

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$set (sort\ xs) = set\ xs$

Better: every  $x$  occurs as often in  $sort\ xs$  as in  $xs$ .

More succinctly:

$mset (sort\ xs) = mset\ xs$

where  $mset :: 'a list \Rightarrow 'a multiset$

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## What are multisets?

Sets with (possibly) repeated elements

Some operations:

$\{\#\}$   $:: 'a multiset$   
 $add\_mset$   $:: 'a \Rightarrow 'a multiset \Rightarrow 'a multiset$   
 $+$   $:: 'a multiset \Rightarrow 'a multiset \Rightarrow 'a multiset$

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 $mset$  :: 'a list  $\Rightarrow$  'a multiset  
 $set\_mset$  :: 'a multiset  $\Rightarrow$  'a set

```
Import "~/src/HOL/Library/Multiset"
```

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- 1 Correctness
- 2 Insertion Sort
- 3 Time
- 4 Merge Sort

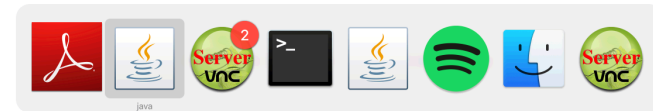
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HOL/Data\_Structures/Sorting.thy

Insertion Sort Correctness

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HOL/Data\_Structures/Sorting.thy



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