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Title: Petter: Compiler Construction (14.05.2020) - 16: Left Recursion and LL(k)

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Left Recursion

Attention:
Many grammars are not $LL(k)$!

A reason for that is:

Definition
Grammar G is called left-recursive, if

$$A \to^+ A\,\beta \qquad \text{for an} \quad A \in N\,,\ \beta \in (T \cup N)^*$$

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Example:

$$\begin{array}{lclcll} E & \to & E + T & \mid & T & \\ T & \to & T * F & \mid & F & \\ F & \to & (\,E\,) & \mid & \text{name} & \mid \ \text{int} \end{array}$$

... is left-recursive

Left Recursion

Theorem:
Let a grammar G be reduced and left-recursive, then G is not $LL(k)$ for any k.

Proof:
Let wlog. $A \to A\,\beta \mid \alpha \quad \in P$
and A be reachable from S

Assumption: G is $LL(k)$

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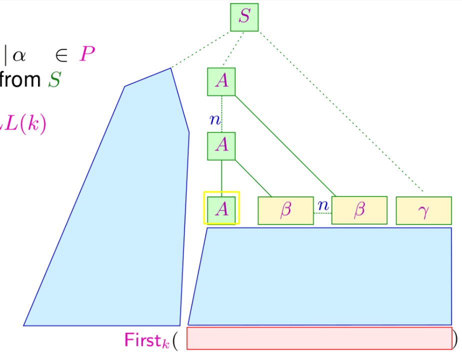

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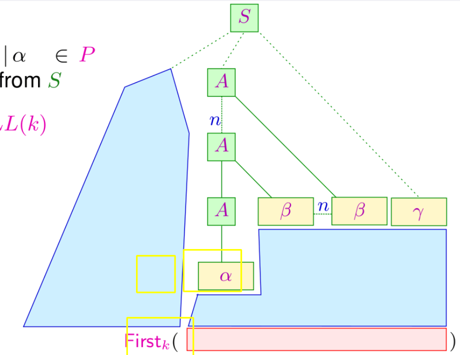

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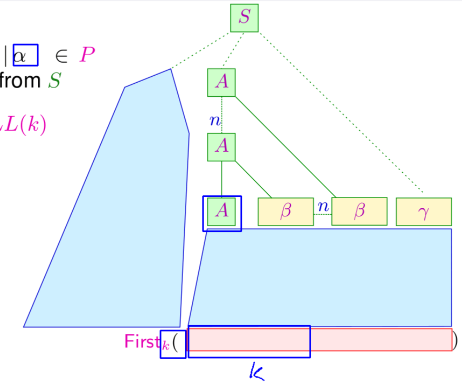

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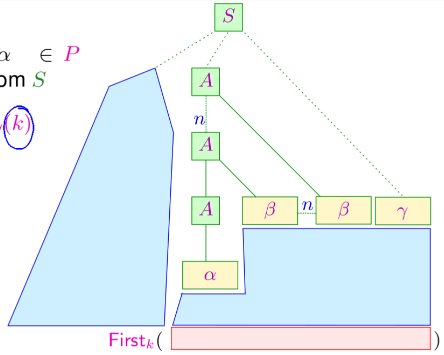

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Case 1: $\beta \to^* \epsilon$ — Contradiction !!!
Case 2: $\beta \to^* w \neq \epsilon \implies \text{First}_k(\alpha\,w^k\,\gamma) \cap \text{First}_k(\alpha\,w^{k+1}\,\gamma) \neq \emptyset$