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Title: Petter: Compiler Construction (23.04.2020) - 04: Naive Berry-Sethi

Date: Wed Apr 22 09:33:14 CEST 2020

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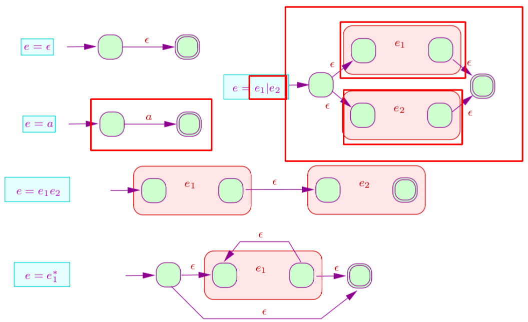
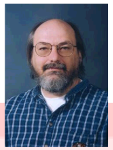
Pages: 21

Chapter 3: Converting Regular Expressions to NFAs

In Linear Time from Regular Expressions to NFAs



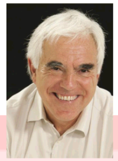


Ken Thompson

Thompson's Algorithm

Produces $\mathcal{O}(n)$ states for regular expressions of length n.

A formal approach to Thompson's Algorithm



Gerard Berry

Ravi Sethi

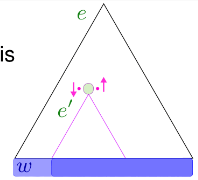
Berry-Sethi Algorithm

Produces exactly $n+1$ states without ϵ-transitions and demonstrates → *Equality Systems* and → *Attribute Grammars*

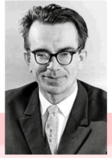
Idea:

An automaton covering the syntax tree of a regular expression e tracks (conceptionally via markers "•"), which subexpressions e' are reachable consuming the rest of input w.

- markers contribute an entry or exit point into the automaton for this subexpression
- edges for each layer of subexpression are modelled after Thompson's automata



A formal approach to Thompson's Algorithm


Viktor M. Glushkov

Glushkov Automaton

Produces exactly $n+1$ states without ϵ-transitions and demonstrates $\rightarrow$ *Equality Systems* and $\rightarrow$ *Attribute Grammars*

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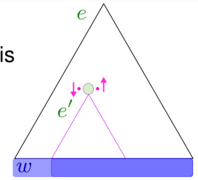

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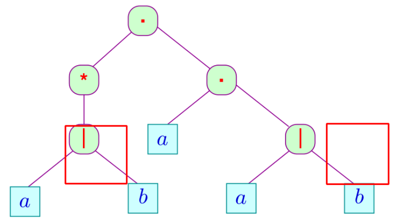
Berry-Sethi Approach

0|1

3|4

... for example:

$(a|b)^*a(a|b)$



Berry-Sethi Approach (naive version)

Construction (naive version):

States: $\boxed{\bullet r}$, $\boxed{r\bullet}$ with r nodes of e;
Start state: $\bullet e$;
Final state: $e\bullet$;
Transitions: for leaves $r \equiv \boxed{i \mid x}$ we require: $(\bullet r, x, r\bullet)$.

The leftover transitions are:

r	Transitions
$r_1 \mid r_2$	$(\bullet r, \epsilon, \bullet r_1)$ $(\bullet r, \epsilon, \bullet r_2)$ $(r_1\bullet, \epsilon, r\bullet)$ $(r_2\bullet, \epsilon, r\bullet)$
$r_1 \cdot r_2$	$(\bullet r, \epsilon, \bullet r_1)$ $(r_1\bullet, \epsilon, \bullet r_2)$ $(r_2\bullet, \epsilon, r\bullet)$

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Berry-Sethi Approach

$\bullet$ 0|1

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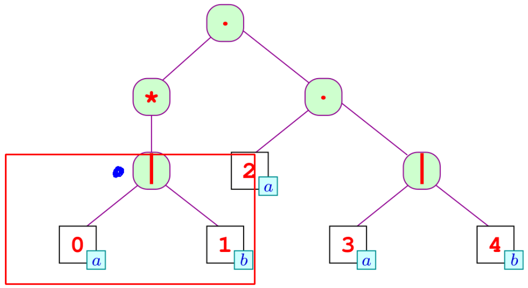

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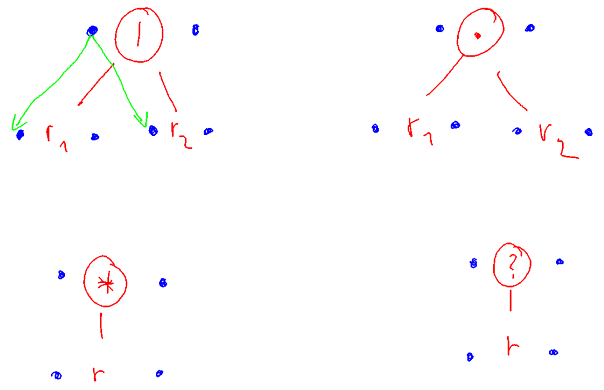

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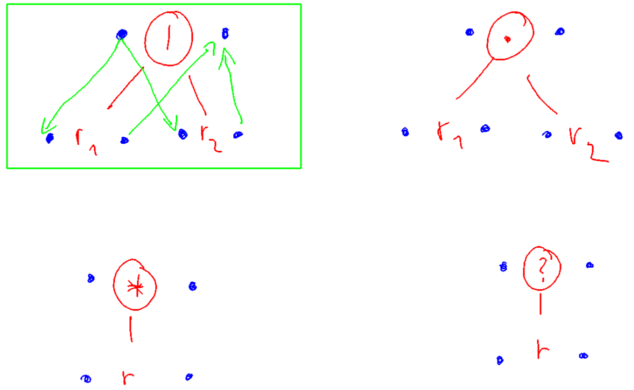

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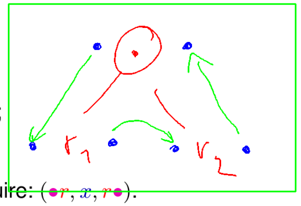

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