Script **generated by TTT**

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The instruction new $(tag, args)$ creates a corresponding object (B, C, F, V) in H and returns a reference to it.

We distinguish three different kinds of code for an expression e:

- $\text{code}_V\ e$ — (generates code that) computes the Value of e, stores it in the heap and returns a reference to it on top of the stack (the normal case);
- $\text{code}_B\ e$ — computes the value of e, and returns it on the top of the stack (only for Basic types);
- $\text{code}_C\ e$ — does not evaluate e, but stores a Closure of e in the heap and returns a reference to the closure on top of the stack.

We start with the code schemata for the first two kinds:

13 Simple expressions

Expressions consisting only of constants, operator applications, and conditionals are translated like expressions in imperative languages:

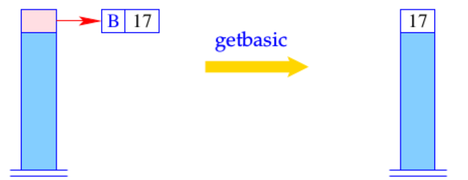
$$
\begin{array}{lcl}
\text{code}_B\ b\ \rho\ \text{sd} & = & \text{loadc b} \\
\text{code}_B\ (\Box_1\ e)\ \rho\ \text{sd} & = & \text{code}_B\ e\ \rho\ \text{sd} \\
 & & \text{op}_1 \\
\text{code}_B\ (e_1\ \Box_2\ e_2)\ \rho\ \text{sd} & = & \text{code}_B\ e_1\ \rho\ \text{sd} \\
 & & \text{code}_B\ e_2\ \rho\ (\text{sd}+1) \\
 & & \text{op}_2
\end{array}
$$

$$
\begin{array}{lcll}
\text{code}_B\ (\textbf{if}\ e_0\ \textbf{then}\ e_1\ \textbf{else}\ e_2)\ \rho\ \text{sd} & = & & \text{code}_B\ e_0\ \rho\ \text{sd} \\
 & & & \text{jumpz A} \\
 & & & \text{code}_B\ e_1\ \rho\ \text{sd} \\
 & & & \text{jump B} \\
 & & \text{A:} & \text{code}_B\ e_2\ \rho\ \text{sd} \\
 & & \text{B:} & \dots
\end{array}
$$

Remark

- ρ denotes the actual address environment, in which the expression is translated.
- The extra argument sd, the stack difference, *simulates* the movement of the SP when instruction execution modifies the stack. It is needed later to address variables.
- The instructions op_1 and op_2 implement the operators $\Box_1$ and $\Box_2$, in the same way as the the operators neg and add implement negation resp. addition in the CMa.
- For all other expressions, we first compute the value in the heap and then dereference the returned pointer:

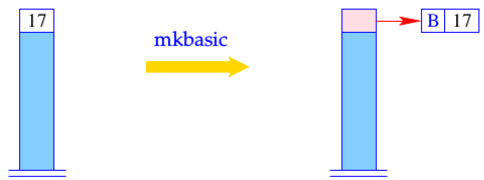
$$\boxed{\text{code}_B\ e\ \rho\ \text{sd}} \quad = \quad \begin{array}{l} \text{code}_V\ e\ \rho\ \text{sd} \\ \text{getbasic} \end{array}$$




```
if (H[S[SP]] != (B,_))
    Error "not basic!";
else
    S[SP] = H[S[SP]].v;
```


For code_V and simple expressions, we define analogously:

$$\begin{array}{lcl}
\text{code}_V\ b\ \rho\ \text{sd} & = & \text{loadc } b;\ \text{mkbasic} \\
\text{code}_V\ (\Box_1\ e)\ \rho\ \text{sd} & = & \text{code}_B\ e\ \rho\ \text{sd} \\
 & & \text{op}_1;\ \text{mkbasic} \\
\text{code}_V\ (e_1\ \Box_2\ e_2)\ \rho\ \text{sd} & = & \text{code}_B\ e_1\ \rho\ \text{sd} \\
 & & \text{code}_B\ e_2\ \rho\ (\text{sd}+1) \\
 & & \text{op}_2;\ \text{mkbasic} \\
\text{code}_V\ (\textbf{if}\ e_0\ \textbf{then}\ e_1\ \textbf{else}\ e_2)\ \rho\ \text{sd} & = & \text{code}_B\ e_0\ \rho\ \text{sd} \\
 & & \text{jumpz A} \\
 & & \text{code}_V\ e_1\ \rho\ \text{sd} \\
 & & \text{jump B} \\
 & \text{A:} & \text{code}_V\ e_2\ \rho\ \text{sd} \\
 & \text{B:} & \ldots
\end{array}$$




```
S[SP] = new (B,S[SP]);
```

For code_V and simple expressions, we define analogously:

$$
\begin{array}{llll}
\text{code}_V\, b\, \rho\, \text{sd} & = & & \text{loadc b; mkbasic} \\
\text{code}_V\, (\Box_1\, e)\, \rho\, \text{sd} & = & & \text{code}_B\, e\, \rho\, \text{sd} \\
 & & & \text{op}_1;\, \mathbf{mkbasic} \\
\text{code}_V\, (e_1\, \Box_2\, e_2)\, \rho\, \text{sd} & = & & \text{code}_B\, e_1\, \rho\, \text{sd} \\
 & & & \text{code}_B\, e_2\, \rho\, (\text{sd}+1) \\
 & & & \text{op}_2;\, \mathbf{mkbasic} \\
\text{code}_V\, (\mathbf{if}\ e_0\ \mathbf{then}\ e_1\ \mathbf{else}\ e_2)\, \rho\, \text{sd} & = & & \text{code}_B\, e_0\, \rho\, \text{sd} \\
 & & & \text{jumpz A} \\
 & & & \text{code}_V\, e_1\, \rho\, \text{sd} \\
 & & & \text{jump B} \\
 & & \text{A:} & \text{code}_V\, e_2\, \rho\, \text{sd} \\
 & & \text{B:} & \ldots
\end{array}
$$

14 Accessing Variables

We must distinguish between local and global variables.

Example Regard the function f :

$$
\begin{array}{lll}
\mathbf{let} & c = 5 & \\
\mathbf{in\ let} & f = \mathbf{fun}\ a \to & \mathbf{let}\ b = a * a \\
 & & \mathbf{in}\ b + c \\
\mathbf{in} & f\ c &
\end{array}
$$

The function f uses the global variable c and the local variables a (as formal parameter) and b (introduced by the inner **let**).

The binding of a global variable is determined, when the function is constructed (static binding!), and later only looked up.

Accessing Global Variables

- The bindings of global variables of an expression or a function are kept in a vector in the heap (Global Vector).
- They are addressed consecutively starting with 0.
- When an F-object or a C-object are constructed, the Global Vector for the function or the expression is determined and a reference to it is stored in the gp-component of the object.
- During the evaluation of an expression, the (new) register GP (Global Pointer) points to the actual Global Vector.
- In constrast, local variables should be administered on the stack ...

$\Longrightarrow$ General form of the address environment:

$$\rho : Vars \to \{L, G\} \times \mathbb{Z}$$

Accessing Local Variables

Local variables are administered on the stack, in stack frames.

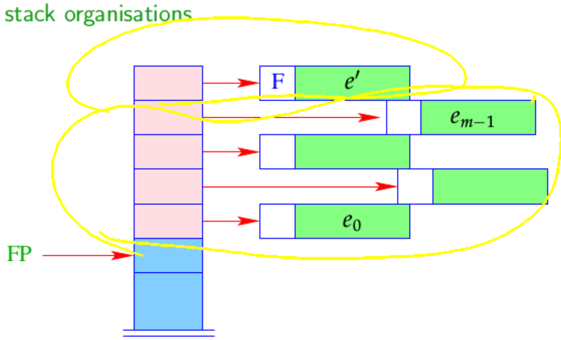
Let $e \equiv e'\ e_0\ \ldots\ e_{m-1}$ be the application of a function e' to arguments $e_0, \ldots, e_{m-1}$.

Caveat

The arity of e' does not need to be m.

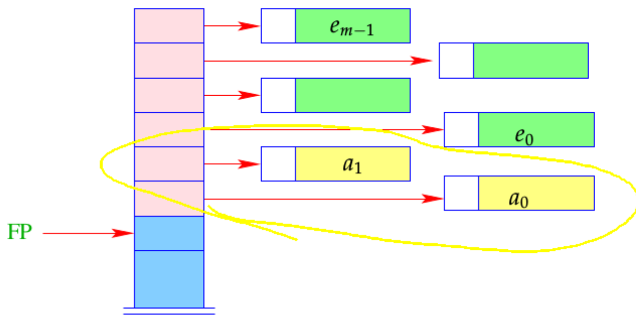
- f may therefore receive less than n arguments (under supply);
- f may also receive more than n arguments, if t is a functional type (over supply).

Possible stack organisations

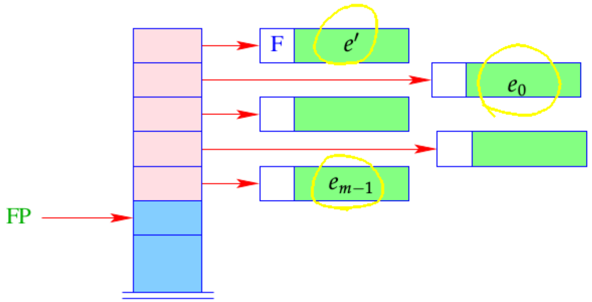


+ Addressing of the arguments can be done relative to FP
– The local variables of e' cannot be addressed relative to FP.
– If e' is an n-ary function with $n < m$, i.e., we have an over-supplied function application, the remaining $m - n$ arguments will have to be shifted.

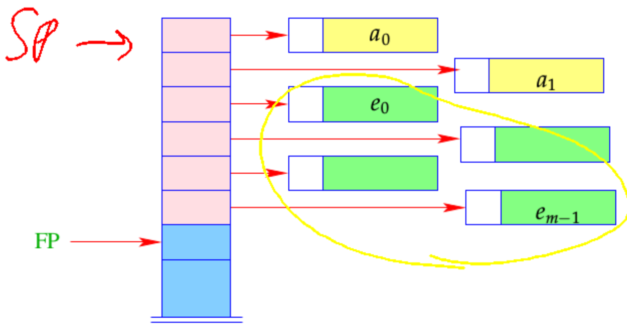
– If e' evaluates to a function, which has already been partially applied to the parameters $a_0, \ldots, a_{k-1}$, these have to be sneaked in underneath e_0:



Alternative



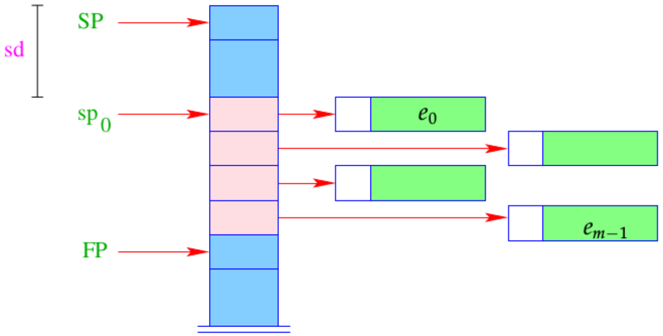
+ The further arguments $a_0, \ldots, a_{k-1}$ and the local variables can be allocated above the arguments.



– Addressing of arguments and local variables relative to FP is no more possible. (Remember: m is unknown when the function definition is translated.)

Way out

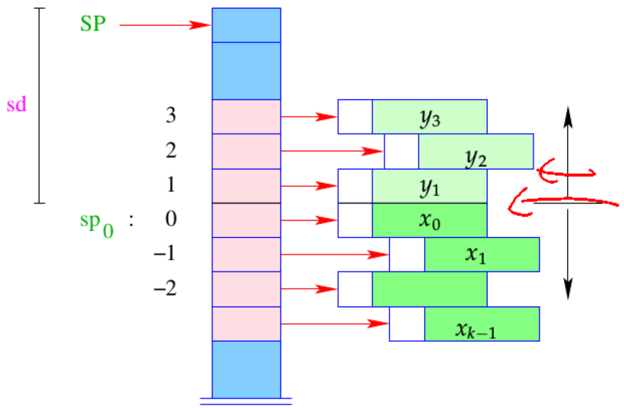
- We address both, arguments and local variables, relative to the stack pointer SP !!!
- However, the stack pointer changes during program execution...



- The differerence between the current value of SP and its value sp_0 at the entry of the function body is called the stack distance, sd.
- Fortunately, this stack distance can be determined at compile time for each program point, by simulating the movement of the SP.
- The formal parameters $x_0, x_1, x_2, \ldots$ successively receive the non-positive relative addresses $0, -1, -2, \ldots$, i.e., $\rho\, x_i = (L, -i)$.
- The absolute address of the i-th formal parameter consequently is

$$sp_0 - i = (SP - sd) - i$$

- The local **let**-variables $y_1, y_2, y_3, \ldots$ will be successively pushed onto the stack:



- The y_i have positive relative addresses $1, 2, 3, \ldots$, that is: $\rho\, y_i = (L, i)$.
- The absolute address of y_i is then $sp_0 + i = (SP - sd) + i$

With CBN, we generate for the access to a variable:

$$\begin{array}{lll} \text{code}_V\ x\ \rho\ \text{sd} & = & \text{getvar}\ x\ \rho\ \text{sd} \\ & & \text{eval} \end{array}$$

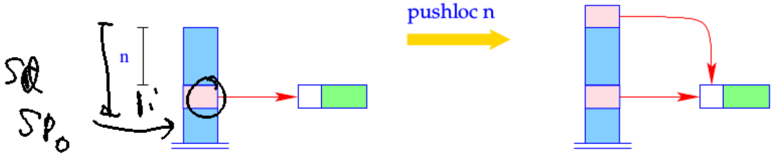
The instruction eval checks, whether the value has already been computed or whether its evaluation has to yet to be done $\Longrightarrow$ will be treated later.

With CBV, we can just delete eval from the above code schema.

The (compile-time) macro getvar is defined by:

$$\begin{array}{lll} \text{getvar}\ x\ \rho\ \text{sd} & = & \textbf{let}\ (t, i) = \rho\ x\ \textbf{in} \\ & & \textbf{match}\ t\ \textbf{with} \\ & & \quad L \to \text{pushloc}\ (\text{sd} - i) \\ & & \quad \mid G \to \text{pushglob i} \\ & & \textbf{end} \end{array}$$

The access to local variables:



S[SP+1] =S[SP - n]; SP++;

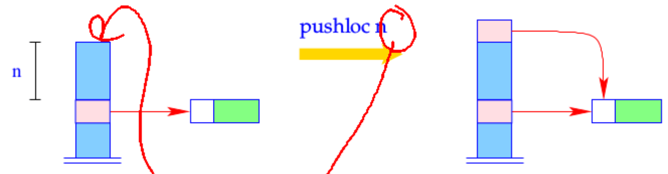
Correctness argument

Let sp and sd be the values of the stack pointer resp. stack distance before the execution of the instruction. The value of the local variable with address i is loaded from $S[a]$ with

$$a = \mathrm{sp} - (\mathrm{sd} - i) = (\mathrm{sp} - \mathrm{sd}) + i = \mathrm{sp}_0 + i$$

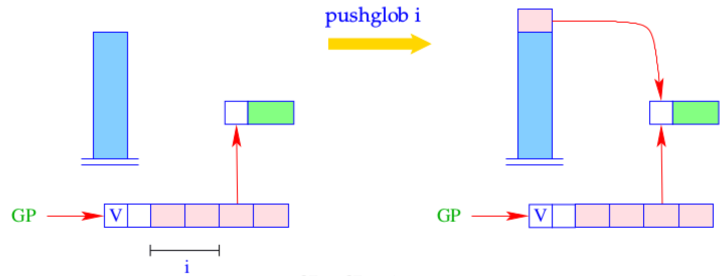
... exactly as it should be.

The access to local variables:



S[SP+1] =S[SP - n]; SP++;

The access to global variables is much simpler:



SP = SP + 1;
S[SP] = GP→v[i];

Example

Regard $e \equiv (b + c)$ for $\rho = \{b \mapsto (L,1), c \mapsto (G,0)\}$ and $\mathrm{sd} = 1$.

With CBN, we obtain:

$\mathrm{code}_V\ e\ \rho\ 1$	$=$	getvar $b\ \rho$ 1	$=$ 1	pushloc 0
		eval	2	eval
		getbasic	2	getbasic
		getvar $c\ \rho$ 2	2	pushglob 0
		eval	3	eval
		getbasic	3	getbasic
		add	3	add
		mkbasic	2	mkbasic

15 let-Expressions

As a warm-up let us first consider the treatment of local variables.

Let $e \equiv \textbf{let}\ y_1 = e_1\ \textbf{in} \ldots \textbf{let}\ y_n = e_n\ \textbf{in}\ e_0$ be a nested **let**-expression.

The translation of e must deliver an instruction sequence that

- allocates local variables $y_1, \ldots, y_n$;
- in the case of
  CBV: evaluates $e_1, \ldots, e_n$ and binds the y_i to their values;
  CBN: constructs closures for the $e_1, \ldots, e_n$ and binds the y_i to them;
- evaluates the expression e_0 and returns its value.

Here, we consider the non-recursive case only, i.e. where y_j only depends on $y_1, \ldots, y_{j-1}$. We obtain for CBN:

$$
\begin{array}{lll}
\mathrm{code}_V\ e\ \rho\ \mathrm{sd} & = & \mathrm{code}_C\ e_1\ \rho\ \mathrm{sd} \\
 & & \mathrm{code}_C\ e_2\ \rho_1\ (\mathrm{sd}+1) \\
 & & \ldots \\
 & & \mathrm{code}_C\ e_n\ \rho_{n-1}\ (\mathrm{sd}+n-1) \\
 & & \mathrm{code}_V\ e_0\ \rho_n\ (\mathrm{sd}+n) \\
 & & \mathrm{slide}\ n \qquad // \text{ deallocates local variables}
\end{array}
$$

e_1 $x = 1$

e_2 $x = x * x$ $x + 2$

e_3 $x = x - 5$

$x + 7$

where $\rho_j = \rho \oplus \{y_i \mapsto (L, \mathrm{sd}+i) \mid i = 1, \ldots, j\}$.

In the case of CBV, we use code_V for the expressions $e_1, \ldots, e_n$.

Caveat!

All the e_i must be associated with the same binding for the global variables!

$\rho_0 = \rho \qquad \rho_{i+1} = \rho_i \oplus \{y_i \mapsto (L, \mathrm{sd}+i)\}$

$$
\begin{array}{lll}
\mathrm{code}_V\ e\ \rho\ \mathrm{sd} & = & \mathrm{code}_C\ e_1\ \rho\ \mathrm{sd} \\
 & & \mathrm{code}_C\ e_2\ \rho_1\ (\mathrm{sd}+1) \\
 & & \ldots \\
 & & \mathrm{code}_C\ e_n\ \rho_{n-1}\ (\mathrm{sd}+n-1) \\
 & & \mathrm{code}_V\ e_0\ \rho_n\ (\mathrm{sd}+n) \\
 & & \mathrm{slide}\ n \qquad // \text{ deallocates local variables}
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$$

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$\mathrm{code}_V\,(e_i)$

Example

Consider the expression

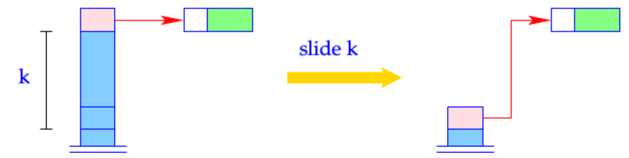
$$e \equiv \textbf{let } a = 19 \textbf{ in let } b = a * a \textbf{ in } a + b$$

for $\rho = \emptyset$ and sd $= 0$. We obtain (for CBV):

0	loadc 19	3	getbasic	3	pushloc 1
1	mkbasic	3	mul	4	getbasic
1	pushloc 0	2	mkbasic	4	add
2	getbasic	2	pushloc 1	3	mkbasic
2	pushloc 1	3	getbasic	3	slide 2

$\{a \mapsto (L,1), b \mapsto (L,2)\}$

The instruction slide k deallocates again the space for the locals:



S[SP-k] = S[SP];
SP = SP - k;

16 Function Definitions

The definition of a function f requires code that allocates a functional value for f in the heap. This happens in the following steps:

- Creation of a Global Vector with the binding of the free variables;
- Creation of an (initially empty) argument vector;
- Creation of an F-Object, containing references to theses vectors and the start address of the code for the body;

Separately, code for the body has to be generated.

Thus,

$$\begin{array}{rrl}
\text{code}_V\,(\textbf{fun } x_0 \ldots x_{k-1} \to e)\,\rho\ \text{sd} \quad = & & \text{getvar } z_0\ \rho\ \text{sd} \\
& & \text{getvar } z_1\ \rho\ (\text{sd}+1) \\
& & \ldots \\
& & \text{getvar } z_{g-1}\ \rho\ (\text{sd}+g-1) \\
& & \text{mkvec g} \\
& & \text{mkfunval A} \\
& & \text{jump B} \\
& \text{A}: & \text{targ k} \\
& & \text{code}_V\ e\ \rho'\ 0 \\
& & \text{return k} \\
& \text{B}: & \ldots
\end{array}$$

where $\{z_0, \ldots, z_{g-1}\} = \mathit{free}(\textbf{fun } x_0 \ldots x_{k-1} \to e)$
and $\rho' = \{x_i \mapsto (L, -i) \mid i = 0, \ldots, k-1\} \cup \{z_j \mapsto (G, j) \mid j = 0, \ldots, g-1\}$

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& & & \mathrm{getvar}\ z_1\ \rho\ (\mathrm{sd}+1) \\
& & & \ldots \\
& & & \mathrm{getvar}\ z_{g-1}\ \rho\ (\mathrm{sd}+g-1) \\
& & & \mathrm{mkvec}\ \mathrm{g} \\
& & & \mathrm{mkfunval}\ \mathrm{A} \\
& & & \mathrm{jump}\ \mathrm{B} \\
& & \mathrm{A}: & \mathrm{targ}\ \mathrm{k} \\
& & & \mathrm{code}_V\ e\ \rho'\ 0 \\
& & & \mathrm{return}\ \mathrm{k} \\
& & \mathrm{B}: & \ldots
\end{array}$$

where $\{z_0, \ldots, z_{g-1}\} = free(\mathbf{fun}\ x_0 \ldots x_{k-1} \to e)$
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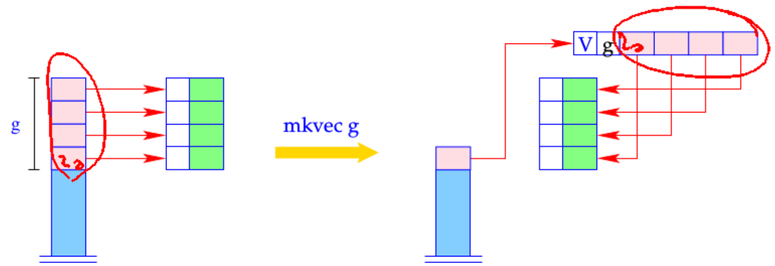


```
h = new (V, n);
SP = SP - g + 1;
for (i=0; i<g; i++)
    h→v[i] = S[SP + i];
S[SP] = h;
```


$$\begin{array}{rlll}
\mathrm{code}_V\ (\mathbf{fun}\ x_0 \ldots x_{k-1} \to e)\ \rho\ \mathrm{sd} & = & & \mathrm{getvar}\ z_0\ \rho\ \mathrm{sd} \\
& & & \mathrm{getvar}\ z_1\ \rho\ (\mathrm{sd}+1) \\
& & & \ldots \\
& & & \mathrm{getvar}\ z_{g-1}\ \rho\ (\mathrm{sd}+g-1) \\
& & & \mathrm{mkvec}\ \mathrm{g} \\
& & & \mathrm{mkfunval}\ \mathrm{A} \\
& & & \mathrm{jump}\ \mathrm{B} \\
& & \mathrm{A}: & \mathrm{targ}\ \mathrm{k} \\
& & & \mathrm{code}_V\ e\ \rho'\ 0 \\
& & & \mathrm{return}\ \mathrm{k} \\
& & \mathrm{B}: & \ldots
\end{array}$$

where $\{z_0, \ldots, z_{g-1}\} = free(\mathbf{fun}\ x_0 \ldots x_{k-1} \to e)$
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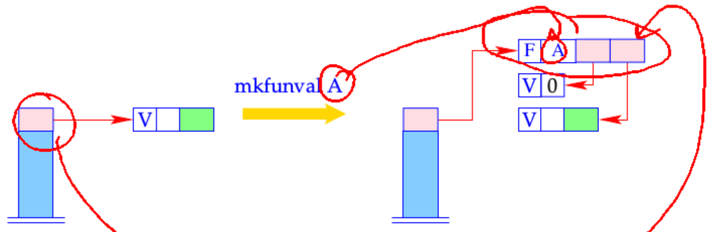

a = new (V,0);
S[SP] = new (F, A, a, S[SP]);

Example

Regard $f \equiv \mathbf{fun}\ b \to a + b$ for $\rho = \{a \mapsto (L, 1)\}$ and sd $= 1$.

code$_V$ f ρ 1 produces:

1		pushloc 0	0	pushglob 0	2		getbasic
2		mkvec 1	1	eval	2		add
2		mkfunval A	1	getbasic	1		mkbasic
2		jump B	1	pushloc 1	1		return 1
0	A :	targ 1	2	eval	2	B :	...

The secrets around targ k and return k will be revealed later.