A program \( p \) is constructed as follows:

\[
\begin{align*}
t & ::= \ a \ | \ X \ | \ _ \ | \ f(t_1, \ldots, t_n) \\
g & ::= p(t_1, \ldots, t_n) \ | \ X = t \\
c & ::= p(X_1, \ldots, X_k) \leftarrow g_1, \ldots, g_r \\
p & ::= c_1, \ldots, c_m ? g
\end{align*}
\]

- A term \( t \) either is an atom, a variable, an anonymous variable or a constructor application.
- A goal \( g \) either is a literal, i.e., a predicate call, or a unification.
- A clause \( c \) consists of a head \( p(X_1, \ldots, X_k) \) with predicate name and list of formal parameters together with a body, i.e., a sequence of goals.
- A program consists of a sequence of clauses together with a single goal as query.

#### A More Realistic Example:

\[
\begin{align*}
\text{app}(X, Y, Z) & \leftarrow X = [], \ Y = Z \\
\text{app}(X, Y, Z) & \leftarrow X = [H, X'], \ Z = [H', Z'], \ \text{app}(X', Y, Z') \\
? \ \text{app}(X, [Y, c], [a, h, Z])
\end{align*}
\]

Remark:

- [\] \quad \text{the atom empty list}
- [\text{[H]} \ Z] \quad \text{binary constructor application}
- [\text{[a, b, Z]}] \quad \text{shortcut for: } [\text{[a} \ | \ [\text{[b]} \ Z] \ | \ ]]]]
A program $p$ is constructed as follows:

$$
t ::= a \mid X \mid f(t_1, \ldots, t_r)
$$

$$
g ::= p(t_1, \ldots, t_k) \mid X = t
$$

$$
c ::= p(X_1, \ldots, X_k) \leftarrow g_1, \ldots, g_r
$$

$$
p ::= c_1 \ldots, c_n g
$$

- A **term** $t$ either is an atom, a variable, an anonymous variable or a constructor application.
- A **goal** $g$ either is a literal, i.e., a predicate call, or a unification.
- A **clause** $c$ consists of a **head** $p(X_1, \ldots, X_k)$ with predicate name and list of formal parameters together with a **body**, i.e., a sequence of goals.
- A **program** consists of a sequence of clauses together with a single goal as query.

---

**Procedural View of Proll programs:**

<table>
<thead>
<tr>
<th>Literal</th>
<th>== procedure call</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicate</td>
<td>== procedure</td>
</tr>
<tr>
<td>Clause</td>
<td>== definition</td>
</tr>
<tr>
<td>Term</td>
<td>== value</td>
</tr>
<tr>
<td>Unification</td>
<td>== basic computation step</td>
</tr>
<tr>
<td>Binding of variables</td>
<td>== side effect</td>
</tr>
</tbody>
</table>

**Note:**

Predicate calls ...

- ... do not have a return value.
- ... affect the caller through side effects only (⇒)
- ... may fail. Then the next definition is tried (⇒)

⇒⇒ backtracking

---

**28 Architecture of the WiM:**

**The Code Store:**

<table>
<thead>
<tr>
<th>C</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
</table>

C = Code store – contains WiM program;

every cell contains one instruction;

PC = Program Counter – points to the next instruction to executed.
The Runtime Stack:

\[ p(X) \leftarrow q(f(X)) \]

S = Runtime Stack – every cell may contain a value or an address;
SP = Stack Pointer – points to the topmost occupied cell;
FP = Frame Pointer – points to the current stack frame.
Frames are created for predicate calls, contain cells for each variable of the current clause.

The Heap:

H = Heap for dynamically constructed terms;
HF = Heap-Pointer – points to the first free cell;

- The heap is maintained like a stack as well :)
- A new-instruction allocates an object in H.
- Objects are tagged with their types (as in the MaMa) ...

29 Construction of Terms in the Heap

Parameter terms of goals (calls) are constructed in the heap before passing.
Assume that the address environment \( \rho \) returns, for each clause variable \( X \) its address (relative to FP) on the stack. Then \( \text{code}_{X} t \rho \) should ...
- construct (a presentation of) \( t \) in the heap; and
- return a reference to it on top of the stack.

Idea:
- Construct the tree during a post-order traversal of \( t \)
- with one instruction for each new node!

Example: \( t \equiv f(g(X, Y), a, Z) \).
Assume that \( X \) is initialized, i.e., \( S[FP + \rho X] \) contains already a reference, \( Y \) and \( Z \) are not yet initialized.
Representing $t \equiv f(g(X, Y), a, Z)$:

For a distinction, we mark occurrences of already initialized variables through over-lining (e.g. $\tilde{X}$).

**Note:** Arguments are always initialized!

Then we define:

\[
\begin{align*}
\text{code}_A \ a \, \rho & = \text{putatom} \ a \\
\text{code}_A \ X \, \rho & = \text{putvar} \ (\rho \ X) \\
\text{code}_A \ X \, \rho & = \text{putref} \ (\rho \ X) \\
\text{code}_A \ _\_ \, \rho & = \text{putanon} \\
\text{code}_A \ f(t_1, \ldots, t_n) \, \rho & = \text{code}_A \ t_1 \, \rho \\
\text{code}_A \ f(t_1, \ldots, t_n) \, \rho & = \text{putstruct} \ f/n
\end{align*}
\]

For $f(g(\tilde{X}, Y), a, Z)$ and $\rho = \{X \mapsto 1, Y \mapsto 2, Z \mapsto 3\}$ this results in the sequence:

\[
\begin{align*}
\text{putref} 1 & \quad \text{putatom} \ a \\
\text{putvar} 2 & \quad \text{putvar} 3 \\
\text{putstruct} g/2 & \quad \text{putstruct} f/3
\end{align*}
\]
The instruction `putatom a` constructs an atom in the heap:

```
SP++; S[SP] = new (A,a);
```

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The instruction `putvar i` introduces a new unbound variable and additionally initializes the corresponding cell in the stack frame:

```
SP = SP + 1;
S[SP] = new (R, HP);
S[FP + i] = S[SP];
```

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The instruction `putanon` introduces a new unbound variable but does not store a reference to it in the stack frame:

```
SP = SP + 1;
S[SP] = new (R, HP);
```

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The instruction `putref i` pushes the value of the variable onto the stack:

```
SP = SP + 1;
S[SP] = deref(S[FP + i]);
```

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The instruction \texttt{putref i} pushes the value of the variable onto the stack:

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{The stack before and after the \texttt{putref i} instruction.}
\end{figure}

\[ SP = SP + 1; \]
\[ S[SP] = \text{deref} S[FP + i]; \]

The auxiliary function \texttt{deref} contracts chains of references:

\begin{verbatim}
ref deref (ref v) {
    if (H[v] == (R, w) \&\& v1 == w) return deref (w);
    else return v;
}
\end{verbatim}

Remarks:

- The instruction \texttt{putref i} does not just push the reference from \texttt{S[FP + i]} onto the stack, but also dereferences it as much as possible.
  \[ \Rightarrow \] maximal contraction of reference chains.

- In constructed terms, references always point to \texttt{smaller} heap addresses. Also otherwise, this will be often the case. Sadly enough, it cannot be guaranteed in general. :-(

30 The Translation of Literals (Goals)

Idea:

- Literals are treated as \texttt{procedure calls}.
- We first allocate a stack frame.
- Then we construct the actual parameters (in the heap)
- ... and store references to these into the stack frame.
- Finally, we jump to the code for the procedure/predicate.
\[
\text{code}_C \ p(t_1, \ldots, t_k) \ \rho = \ \text{mark B} \quad // \text{allocates the stack frame}
\]
\[
\text{code}_A \ t_1 \ \rho
\]
\[
\ldots
\]
\[
\text{code}_A \ t_k \ \rho
\]
\[
\text{call p}/k \quad // \text{calls the procedure p}/k
\]
\[\text{B: } \ldots\]

Example:
\[
p(a, X, g(X, Y)) \quad \text{with } \rho = \{ X \rightarrow 1, Y \rightarrow 2 \}
\]

We obtain:
\[
\text{mark B}
\]
\[
\text{putref 1}
\]
\[
\text{putatom a}
\]
\[
\text{putvar 1}
\]
\[
\text{putstruct g}/2
\]
\[
\text{call p}/3
\]
\[\text{B: } \ldots\]

Stack Frame of the WiM:

![Stack Frame Diagram]

Remarks:

- The positive continuation address records where to continue after successful treatment of the goal.
- Additional organizational cells are needed for the implementation of backtracking.

will be discussed at the translation of predicates.
31 Unification

Convention:

- By \( \bar{x} \), we denote an occurrence of \( X \);
  it will be translated differently depending on whether the variable is
  initialized or not.
- We introduce the macro \( \text{put} \bar{x} \rho \) :

\[
\begin{align*}
\text{put} \bar{x} \rho &= \text{putvar} (\rho X) \\
\text{put} \_ \rho &= \text{putanon} \\
\text{put} \bar{x} \_ \rho &= \text{putref} (\rho X)
\end{align*}
\]

Let us translate the unification \( \bar{x} = t \).

Idea 1:

- Push a reference to (the binding of) \( X \) onto the stack;
- Construct the term \( t \) in the heap;
- Invent a new instruction implementing the unification \( \bar{x} = t \).
Let us translate the unification $\bar{X} = t$.

**Idea 1:**
- Push a reference to (the binding of) $\bar{X}$ onto the stack;
- Construct the term $t$ in the heap;
- Invent a new instruction implementing the unification $\Rightarrow$

\[
\text{code: } (\bar{X} = t) \rho = \text{put } \bar{X} \rho \quad \text{code: } t \rho \quad \text{unify}
\]

---

**Example:**

Consider the equation:

$$\bar{U} = f(\bar{g}(\bar{X}, Y), a, Z)$$

Then we obtain for an address environment

$$\rho = \{ \bar{X} \mapsto 1, Y \mapsto 2, Z \mapsto 3, U \mapsto 4 \}$$

\[
\begin{array}{llll}
\text{putref 4} & \text{putref 1} & \text{putatom a} & \text{unify} \\
\text{putvar 2} & \text{putvar 3} & \text{putstruct g/2} & \text{putstruct f/3}
\end{array}
\]

---

**The Function** $\text{unify()}$

- ... takes two heap addresses.
  - For each call, we guarantee that these are **maximally de-referenced**.
- ... checks whether the two addresses are already **identical**.
  - If so, does nothing $\Rightarrow$
- ... binds **younger variables** (larger addresses) to **older variables** (smaller addresses);
- ... when binding a variable to a term, checks whether the variable occurs inside the term $\Rightarrow$ **occur-check**;
- ... records newly created bindings;
- ... may **fail**. Then **backtracking** is initiated.

The instruction **unify** calls the run-time function **unify()** for the topmost two references:

```
unify (S[SP-1], S[SP]);
SP = SP-2;
```
The instruction `unify` calls the run-time function `unify()` for the topmost two references:

```c
bool unify (ref u, ref v) {
    if (u == v) return true;
    if (H[u] == (R,_,)) {
        if (H[v] == (R,_,)) {
            if (u>v) {
                H[u] = (R,v); trail (u); return true;
            } else {
                H[v] = (R,u); trail (v); return true;
            }
        } elseif (check (u,v)) {
            H[u] = (R,v); trail (u); return true;
        } else {
            backtrack(); return false;
        }
    } ...
    ...
```

...  

The Function `unify()`

- ... takes two heap addresses. For each call, we guarantee that these are maximally de-referenced.
- ... checks whether the two addresses are already identical. If so, does nothing :-)
- ... binds younger variables (larger addresses) to older variables (smaller addresses);
- ... when binding a variable to a term, checks whether the variable occurs inside the term → occur-check;
- ... records newly created bindings;
- ... may fail. Then backtracking is initiated.

```c
...  
    if (H[v] == (R,_,)) {  
        if (check (v,u)) {  
            H[v] = (R,u); trail (v); return true;
        } else {  
            backtrack(); return false;
        }
    }
    if (H[a]==(A,a) && H[v]==(A,a)) return true;
    if (H[a]==(S, f/n) && H[v]==(S, f/n)) {  
        for (int i=1; i<n; i++)  
            if (!unify (deref (H[u+i]), deref (H[v+i]))) return false;
        else  
            backtrack(); return false;
    }
```
• The run-time function `trail()` records the a potential new binding.
• The run-time function `backtrack()` initiates backtracking.
• The auxiliary function `check()` performs the occur-check: it tests whether a variable (the first argument) occurs inside a term (the second argument).
• Often, this check is skipped, i.e.,

```c
bool check (ref u, ref v) { return true; }
```