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Title: Seidl: Programmoptimierung (11.01.2012)

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Warning:

- Our representation of numbers is not unique: 011101 should be accepted iff every word from $011101 \cdot 0^*$ is accepted!
- This property is preserved by union, intersection and complement :-)
- It is lost by projection !!!

$\Longrightarrow$ The automaton for projection must be enriched such that the property is re-established !!

$$\boxed{5x} + \boxed{17y} = 9$$

$$\rightarrow \boxed{x + y = 5}$$

$$\rightarrow \boxed{x + y = z}$$

$$x = 5$$

$$x \geq 20$$

$$x = 20 + 5$$

Observation:

The set of satisfying variable assignments is regular :-))

$\phi_1 \wedge \phi_2$	$\Longrightarrow$	$\mathcal{L}(\phi_1) \cap \mathcal{L}(\phi_2)$	(Intersection)
$\neg\phi$	$\Longrightarrow$	$\overline{\mathcal{L}(\phi)}$	(Complement)
$\exists\, x : \phi$	$\Longrightarrow$	$\pi_1(\mathcal{L}(\phi))$	(Projection)

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Projecting away the x-component:

213	t	1	0	1	0	1	0	1	1	0	0
42	z	0	1	0	1	0	1	0	0	0	0
89	y	1	0	0	1	1	0	1	0	0	0
17	x	1	0	0	0	1	0	0	0	0	0

Projecting away the x-component:

213	t	1	0	1	0	1	0	1	1
42	z	0	1	0	1	0	1	0	0
89	y	1	0	0	1	1	0	1	0

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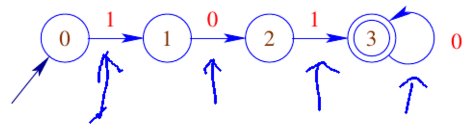
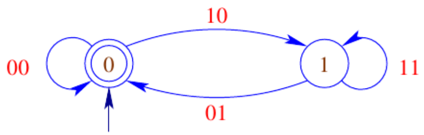
Automata for Basic Predicates:

$x = 5$



Automata for Basic Predicates:

$x+x=y$



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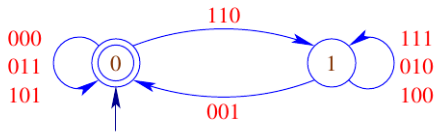

Results:

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3.3 Improving the Memory Layout

Goal:

- Better utilization of caches

  $\Longrightarrow$ reduction of the number of cache misses
- Reduction of allocation/de-allocation costs

  $\Longrightarrow$ replacing heap allocation by stack allocation

  $\Longrightarrow$ support to free superfluous heap objects
- Reduction of access costs

  $\Longrightarrow$ short-circuiting indirection chains (Unboxing)

1. Cache Optimization:

Idea: local memory access

- Loading from memory fetches not just one byte but fills a complete cache line.
- Access to neighbored cells become cheaper.
- If all data of an inner loop fits into the cache, the iteration becomes maximally memory-efficient ...

Possible Solutions:

$\rightarrow$ Reorganize the data accesses !

$\rightarrow$ Reorganize the data !

Such optimizations can be made fully automatic only for arrays :-(

Example:

```
for (j = 1; j < n; j++)
    for (i = 1; i < m; i++)
        a[i][j] = a[i-1][j-1] + a[i][j];
```

⟹ At first, always iterate over the rows!

⟹ Exchange the ordering of the iterations:

for $(i = 1; i < m; i{+}{+})$

  for $(j = 1; j < n; j{+}{+})$

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When is this permitted???

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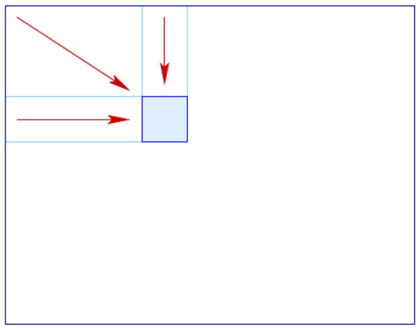
Iteration Scheme: allowed dependencies:



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In our case, we must check that the following equation systems have no solution:

Write		Read
(i_1, j_1)	$=$	$(i_2 - 1, j_2 - 1)$
i_1	$\leq$	i_2
j_2	$\leq$	j_1
(i_1, j_1)	$=$	$(i_2 - 1, j_2 - 1)$
i_2	$\leq$	i_1
j_1	$\leq$	j_2

The first implies: $j_2 \leq j_2 - 1$ Hurra!

The second implies: $i_2 \leq i_2 - 1$ Hurra!

Example: Matrix-Matrix Multiplication

$$
\begin{array}{l}
\text{for } (i = 0; i < N; i{+}{+}) \\
\quad \text{for } (j = 0; j < M; j{+}{+}) \\
\quad\quad \text{for } (k = 0; k < K; k{+}{+}) \\
\quad\quad\quad c[i][j] = c[i][j] + a[i][k] \cdot b[k][j];
\end{array}
$$

?

Over $b[][]$ the iteration is columnwise :-(

Exchange the two inner loops:

$$
\begin{array}{l}
\text{for } (i = 0; i < N; i{+}{+}) \\
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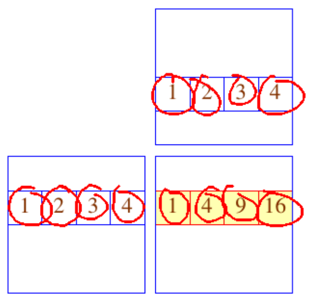
Is this permitted ???



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for $(i = 0; i < N; i{+}{+})$

  for $(k = 0; k < K; k{+}{+})$

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      $c[i][j] = c[i][j] + a[i][k] \cdot b[k][j];$

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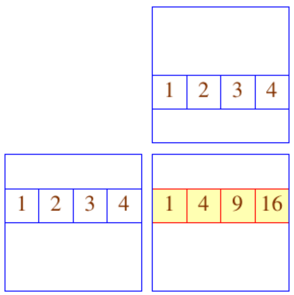

Discussion:

- Correctness follows as before :-)
- A similar idea can also be used for the implementation of multiplication for row compressed matrices :-))
- Sometimes, the program must be massaged such that the transformation becomes applicable :-(
- Matrix-matrix multiplication perhaps requires initialization of the result matrix first ...

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  for $(k = 0; k < K; k{+}{+})$
    for $(j = 0; j < M; j{+}{+})$
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for $(i = 0; i < N; i{+}{+})$
  for $(j = 0; j < M; j{+}{+})$ {
    $c[i][j] = 0;$
    for $(k = 0; k < K; k{+}{+})$
      $c[i][j] = c[i][j] + a[i][k] \cdot b[k][j];$
  }

- Now, the two iterations can no longer be exchanged :-(
- The iteration over j, however, can be duplicated ...

for $(i = 0; i < N; i{+}{+})$ {
  for $(j = 0; j < M; j{+}{+})$ $c[i][j] = 0;$
  for $(j = 0; j < M; j{+}{+})$
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Correctness:

$\Longrightarrow$ The read entries (here: no) may not be modified in the remaining body of the loop !!!

$\Longrightarrow$ The ordering of the write accesses to a memory cell may not be changed :-)

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\begin{array}{l}
\textsf{for}\ \ (i = 0; i < N; i{+}{+}) \\
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Discussion:

- Instead of fusing several loops, we now have distributed the loops :-)
- Accordingly, conditionals may be moved out of the loop $\Longrightarrow$ if-distribution ...

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Idea:

If we find heavily used array elements $a[e_1] \ldots [e_r]$ whose index expressions stay constant within the inner loop, we could instead also provide auxiliary registers :-)

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- Cache-aware organization of other data-structures is possible, but in general not fully automatic ...

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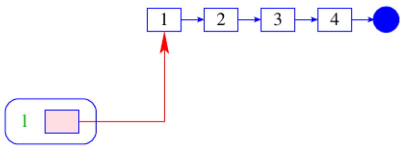

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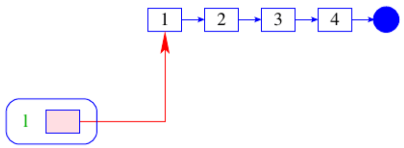

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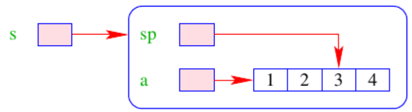
Alternative:



Advantage:

+ The implementation is also simple :-)
+ The operations push / pop still require constant time :-)
+ The data are consequtively allocated; stack oscillations are typically small

$\Longrightarrow$ better Cache behavior !!!

Discussion:

→ The same idea also works for queues :-)

→ Other data-structures are attempted to organize blockwise.

Problem: how can accesses be organized such that they refer mostly to the same block ???

$\Longrightarrow$ Algorithms for external data

2. Stack Allocation instead of Heap Allocation

Problem:

- Programming languages such as Java allocate all data-structures in the heap — even if they are only used within the current method :-(
- If no reference to these data survives the call, we want to allocate these on the stack :-)

$\Longrightarrow$ Escape Analysis

Idea:

Determine points-to information.

Determine if a created object is possibly reachable from the out side ...

Example: Our Pointer Language

$$x = \mathsf{new}();$$
$$y = \mathsf{new}();$$
$$x[A] = y;$$
$$z = y;$$
$$\mathsf{ret} = z;$$

... could be a possible method body ;-)

Accessible from the outside world are memory blocks which:

- are assigned to a global variable such as ret; or
- are reachable from global variables.

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$$x[A] = y;$$
$$z = \boxed{y};$$
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We conclude:

- The objects which have been allocated by the first new() may never escape.
- They can be allocated on the stack :-)

Warning:

This is only meaningful if only few such objects are allocated during a method call :-(

If a local new() occurs within a loop, we still may allocate the objects in the heap ;-)

Extension: Procedures

- We require an interprocedural points-to analysis :-)
- We know the whole program, we can, e.g., merge the control-flow graphs of all procedures into one and compute the points-to information for this.
- Warning: If we always use the same global variables $y_1, y_2, \ldots$ for (the simulation of) parameter passing, the computed information is necessarily imprecise :-(
- If the whole program is not known, we must assume that each reference which is known to a procedure escapes :-((