proof (prove)
g oal (1 subgoal):
1. \( \text{path} \ 0 \ u \ p1 \ w1 \ \& \ \& \ w \ \& \ \& \ (\text{path} \ 0 \ w \ p2 \ w1 \ \& \ w \ \& \ \& \ \text{path} \ 0 \ w \ p3 \ w) \Rightarrow f o o \)
proof (prove)

\begin{itemize}
\item \textbf{goal (1 subgoal)}:
\begin{enumerate}
\item \textbf{hypothesis}: path $G \cup p_1 w_1 \wedge w_2 \wedge w_3$ \\
\textbf{goal}: path $G \cup p_2 w_2 \wedge w_3 \wedge$ path $G \cup p_3 w \Rightarrow fff$
\end{enumerate}
\end{itemize}
shows \( |p'| \cdot \text{length } p > \text{length } p \wedge \neg \text{distinct } p' \wedge \text{path } G \cup p' \cdot v \)

proof:
- \text{proof (state)}
  - \text{this: } \text{length } p < \text{length } p' \wedge \neg \text{distinct } p' \wedge \text{path } G \cup p' \cdot v
- \text{goal: }

\[ \text{no subgoals!} \]
Show that the function defined by \( a \circ 0 = 0 \), and \( a \circ (n+1) = (a \circ n) + 1 \), is bounded by the double-exponential function \( 2 \uparrow\uparrow n \).

**Hint:** We have given you a proof skeleton, setting up the induction. To complete your proof, you should come up with a chain of inequalities. You may try to solve the intermediate steps with a sledgehammer.

**Example:** It is a bit tricky to get the approximation right.

We strongly recommend to sketch the inequalities on paper first.

**Hint:** Have a look at the lemma \( \text{thm} \circ \text{power_even} \), in particular its instance for squares:

\[
\text{thm power_even[where n=2]}
\]

\[
\begin{align*}
\text{lemma} & \quad \text{a n \leq 2 \uparrow (2 \uparrow n)} \quad \text{[1]} \\
\text{proof}[\text{induction n}] & \quad \text{case 0 thus \text{false} by \text{simp}} \\
\text{next} & \quad \text{case (Suc n)} \\
\text{assume} & \quad \text{H: "a n \leq 2 \uparrow (2 \uparrow n - 1)"}
\end{align*}
\]

**proof (prove)**

**goal (1 subgoal):**

1. \( a n \leq 2 \uparrow (2 \uparrow n - 1) \)