Script generated by TTT

Title: Petter: Compilerbau (18.04.2016)

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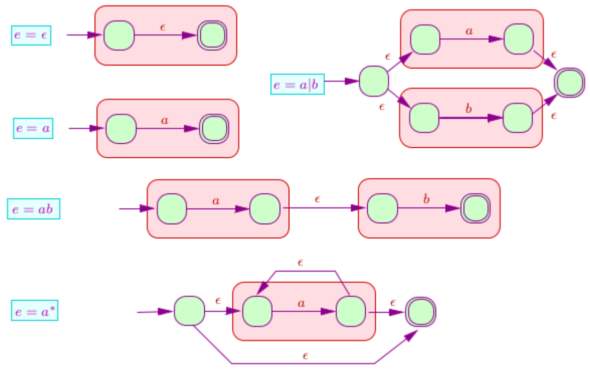
In Linear Time from Regular Expressions to NFAs




Ken Thompson

Thompson's Algorithm

Produces $\mathcal{O}(n)$ states for regular expressions of length n.

Berry-Sethi Approach


Viktor M. Glushkov

Glushkov Algorithm

Produces exactly $n + 1$ states without ϵ-transitions and demonstrates → *Equality Systems* and → *Attribute Grammars*

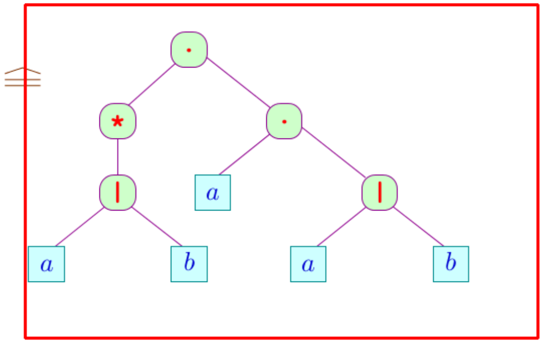
Idea:

The automaton tracks (conceptionally via a marker "•"), in the syntax tree of a regular expression, which subexpressions in e are reachable consuming the rest of input w.

Berry-Sethi Approach

... for example:

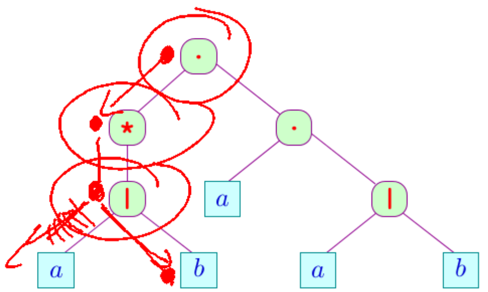
$(a|b)^* a (a|b)$ ⇌



Berry-Sethi Approach

... for example:

$w = bbaa$:



Berry-Sethi Approach

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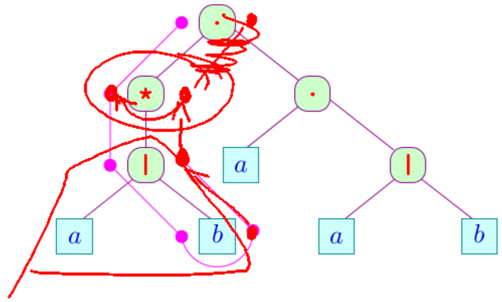
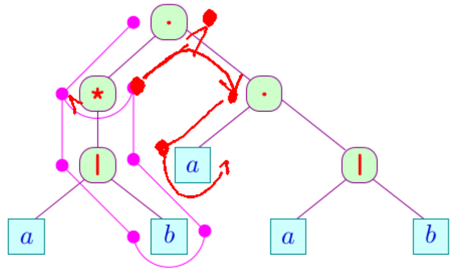


Berry-Sethi Approach

... for example:

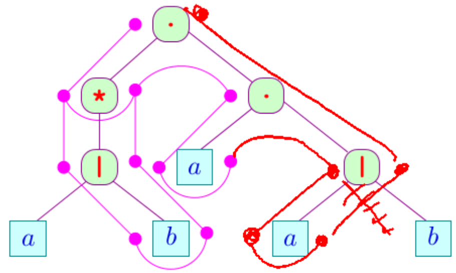
$w = ba$:



Berry-Sethi Approach

... for example:

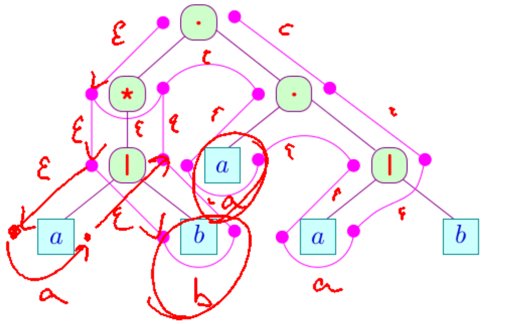
$w = a$:



Berry-Sethi Approach

... for example:

$w \; = \quad :$



Berry-Sethi Approach

In general:

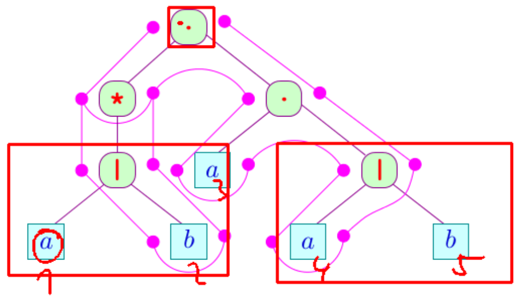
- Input is only consumed at the leaves.
- Navigating the tree does not consume input $\rightarrow$ ϵ-transitions
- For a formal construction we need identifiers for states.
- For a node n's identifier we take the subexpression, corresponding to the subtree dominated by n.
- There are possibly identical subexpressions in one regular expression.

$\Longrightarrow$ we enumerate the leaves ...

Berry-Sethi Approach

... for example:

$w \; = \quad :$



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Berry-Sethi Approach

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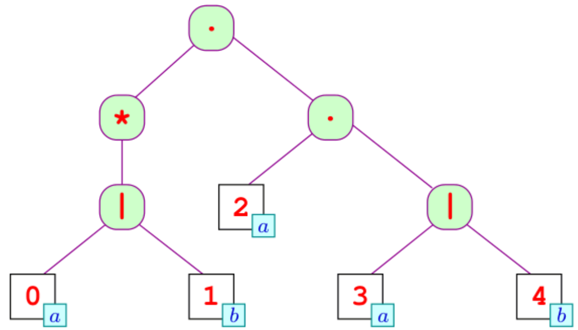


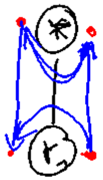
Berry-Sethi Approach (naive version)

Construction (naive version):

States: $\bullet r$, $r\bullet$ with r nodes of e;
Start state: $\bullet e$;
Final state: $e\bullet$;
Transitions: for leaves $r \equiv$ | i | x | we require: $(\bullet r, x, r\bullet)$.

The leftover transitions are:

r	Transitions
$r_1 \mid r_2$	$(\bullet r, \epsilon, \bullet r_1)$ $(\bullet r, \epsilon, \bullet r_2)$ $(r_1\bullet, \epsilon, r\bullet)$ $(r_2\bullet, \epsilon, r\bullet)$
$r_1 \cdot r_2$	$(\bullet r, \epsilon, \bullet r_1)$ $(r_1\bullet, \epsilon, \bullet r_2)$ $(r_2\bullet, \epsilon, r\bullet)$



r	Transitions
r_1^*	$(\bullet r, \epsilon, r\bullet)$ $(\bullet r, \epsilon, \bullet r_1)$ $(r_1\bullet, \epsilon, \bullet r_1)$ $(r_1\bullet, \epsilon, r\bullet)$
$r_1?$	$(\bullet r, \epsilon, r\bullet)$ $(\bullet r, \epsilon, \bullet r_1)$ $(r_1\bullet, \epsilon, r\bullet)$

Berry-Sethi Approach

Discussion:

- Most transitions navigate through the expression
- The resulting automaton is in general nondeterministic

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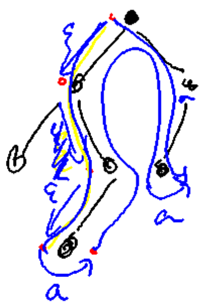
⇒ Strategy for the sophisticated version:
Avoid generating ϵ-transitions



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Pre-compute helper attributes during D(epth)F(irst)S(earch)!

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- Most transitions navigate through the expression
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Pre-compute helper attributes during D(epth)F(irst)S(earch)!

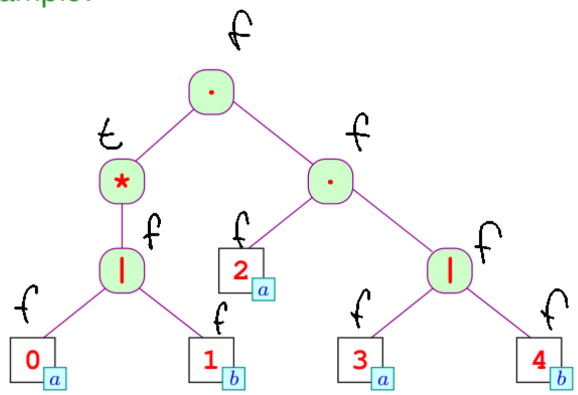
Necessary node-attributes:

- **first** the set of read states below r, which may be reached first, when descending into r.
- **next** the set of read states to the right of r, which may be reached first in the traversal after r.
- **last** the set of read states below r, which may be reached last when descending into r.
- **empty** can the subexpression r consume ϵ ?

Berry-Sethi Approach: 1st step

empty$[r] = t$ if and only if $\epsilon \in [\![r]\!]$

... for example:



Berry-Sethi Approach: 1st step

Implementation:

DFS post-order traversal

for leaves $r \equiv \boxed{i \mid x}$ we find empty$[r] = (x \equiv \epsilon)$.

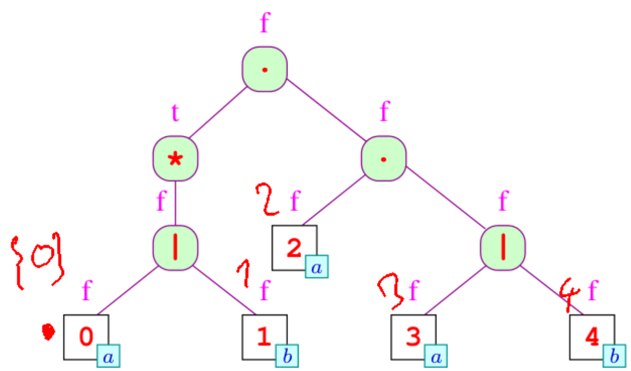
Otherwise:

$$
\begin{array}{lll}
\text{empty}[r_1 \mid r_2] & = & \text{empty}[r_1] \vee \text{empty}[r_2] \\
\text{empty}[r_1 \cdot r_2] & = & \text{empty}[r_1] \wedge \text{empty}[r_2] \\
\text{empty}[r_1^*] & = & t \\
\text{empty}[r_1?] & = & t
\end{array}
$$

Berry-Sethi Approach: 2nd step

The may-set of first reached read states: The set of read states, that may be reached from $\bullet r$ (i.e. while descending into r) via sequences of ϵ-transitions: first$[r] = \{i \text{ in } r \mid (\bullet r, \epsilon, \bullet\boxed{i \mid x}) \in \delta^*, x \neq \epsilon\}$

... for example:



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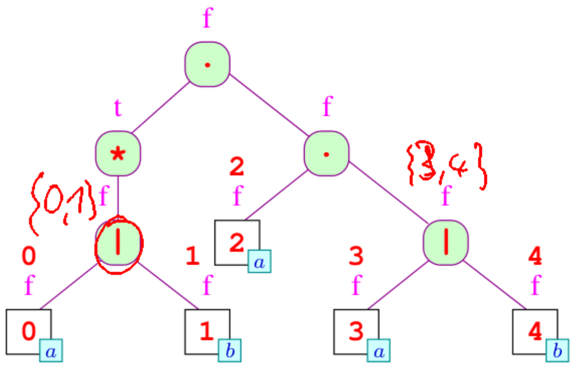

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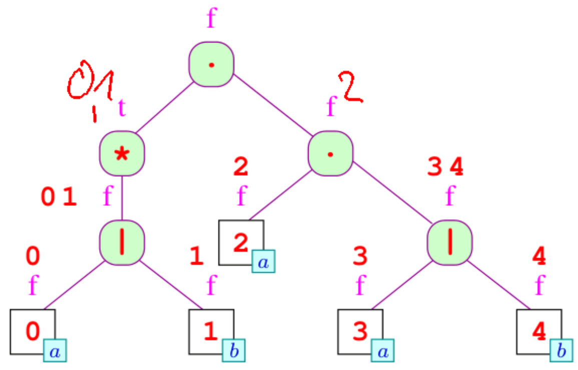

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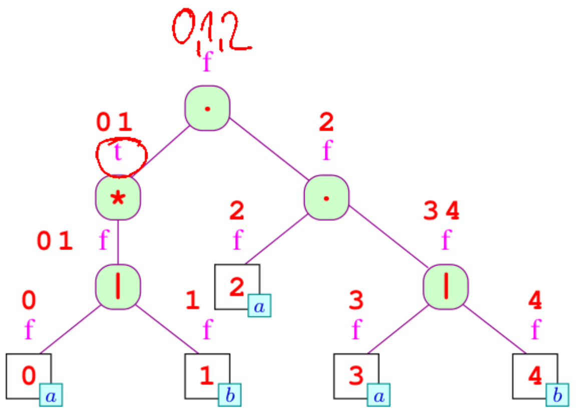

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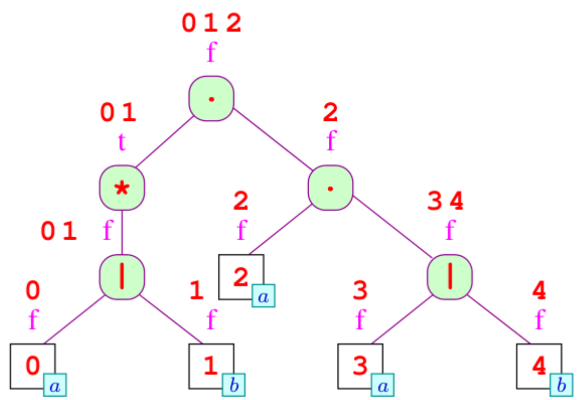


Berry-Sethi Approach: 2nd step

Implementation:
DFS post-order traversal

for leaves $r \equiv \boxed{i \mid x}$ we find $\text{first}[r] = \{i \mid x \neq \epsilon\}$.

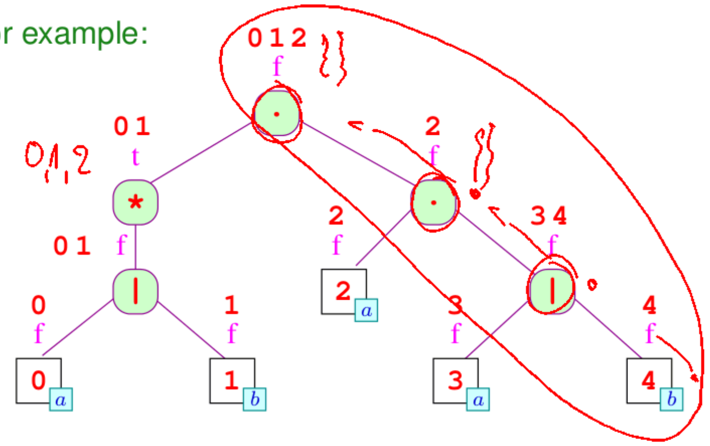
Otherwise:

$$\begin{aligned}
\text{first}[r_1 \mid r_2] &= \text{first}[r_1] \cup \text{first}[r_2] \\
\text{first}[r_1 \cdot r_2] &= \begin{cases} \text{first}[r_1] \cup \text{first}[r_2] & \text{if } \text{empty}[r_1] = t \\ \text{first}[r_1] & \text{if } \text{empty}[r_1] = f \end{cases} \\
\text{first}[r_1^*] &= \text{first}[r_1] \\
\text{first}[r_1?] &= \text{first}[r_1]
\end{aligned}$$

Berry-Sethi Approach: 3rd step

The may-set of next read states: The set of read states within the subtrees right of $r\bullet$, that may be reached next via sequences of ϵ-transitions. $\text{next}[r] = \{i \mid (r\bullet, \epsilon, \bullet\boxed{i \mid x}) \in \delta^*, x \neq \epsilon\}$

... for example:



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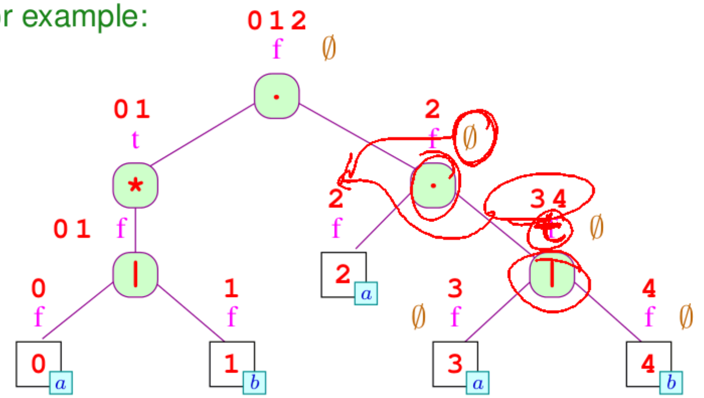

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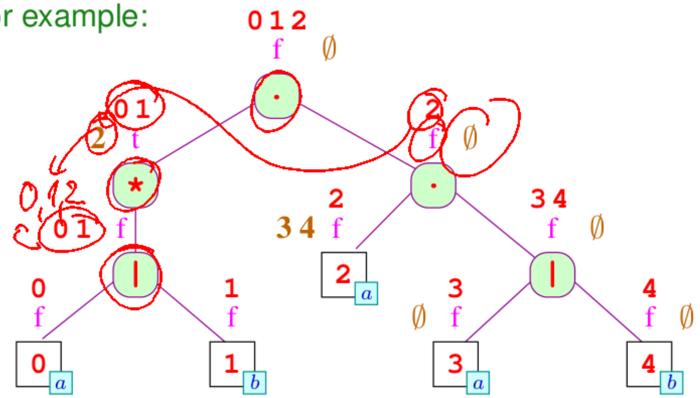


Berry-Sethi Approach: 3rd step

Implementation:

DFS pre-order traversal

For the root, we find: next$[e] = \emptyset$

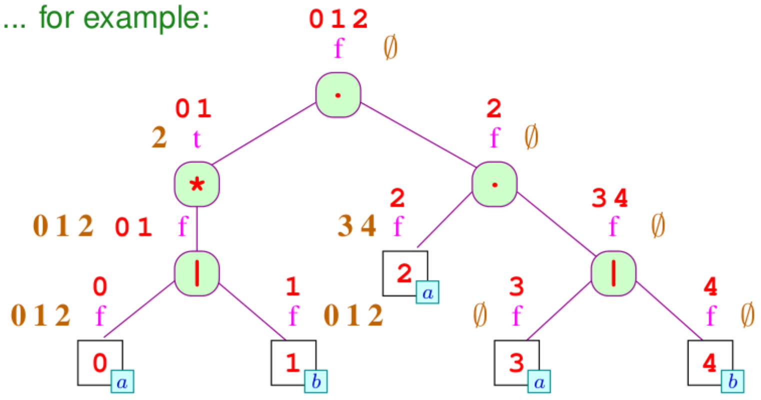
Apart from that we distinguish, based on the context:

r	Equalities
$r_1 \mid r_2$	next$[r_1]$ = next$[r]$ next$[r_2]$ = next$[r]$
$r_1 \cdot r_2$	next$[r_1]$ = $\begin{cases} \text{first}[r_2] \cup \text{next}[r] & \text{if empty}[r_2] = t \\ \text{first}[r_2] & \text{if empty}[r_2] = f \end{cases}$ next$[r_2]$ = next$[r]$
r_1^*	next$[r_1]$ = first$[r_1] \cup$ next$[r]$
$r_1?$	next$[r_1]$ = next$[r]$

Berry-Sethi Approach: 4th step

The may-set of last reached read states: The set of read states, which may be reached last during the traversal of r connected to the root via ϵ-transitions only: last$[r] = \{i \text{ in } r \mid (\boxed{i \mid x}\bullet, \epsilon, r\bullet) \in \delta^*, x \neq \epsilon\}$

... for example:



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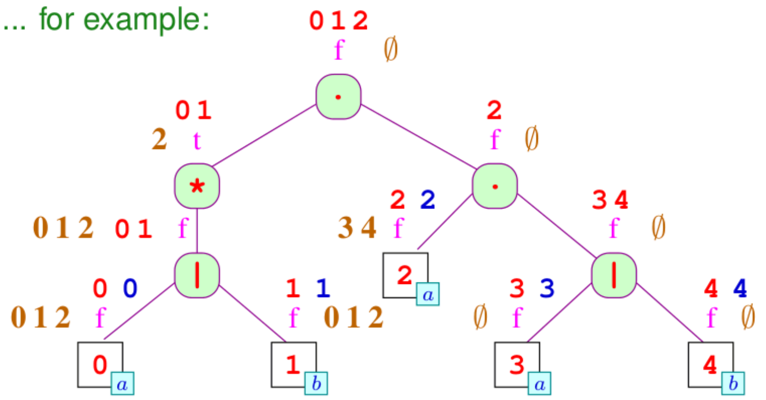

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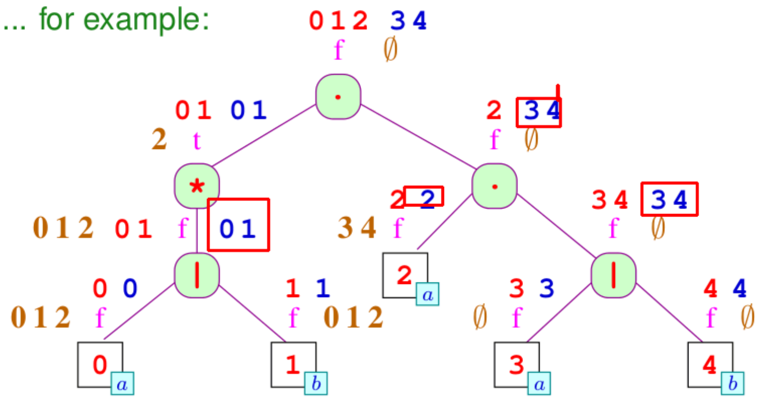


Berry-Sethi Approach: 4th step

Implementation:
DFS post-order traversal

for leaves $r \equiv \boxed{i \mid x}$ we find $\text{last}[r] = \{i \mid x \neq \epsilon\}$.

Otherwise:

$$\text{last}[r_1 \mid r_2] = \text{last}[r_1] \cup \text{last}[r_2]$$

$$\text{last}[r_1 \cdot r_2] = \begin{cases} \text{last}[r_1] \cup \text{last}[r_2] & \text{if } \text{empty}[r_2] = t \\ \text{last}[r_2] & \text{if } \text{empty}[r_2] = f \end{cases}$$

$$\text{last}[r_1^*] = \text{last}[r_1]$$

$$\text{last}[r_1?] = \text{last}[r_1]$$

Berry-Sethi Approach: (sophisticated version)

Construction (sophisticated version):
Create an automanton based on the syntax tree's new attributes:

States: $\{\bullet e\} \cup \{i\bullet \mid i \text{ a leaf}\}$

Start state: $\bullet e$

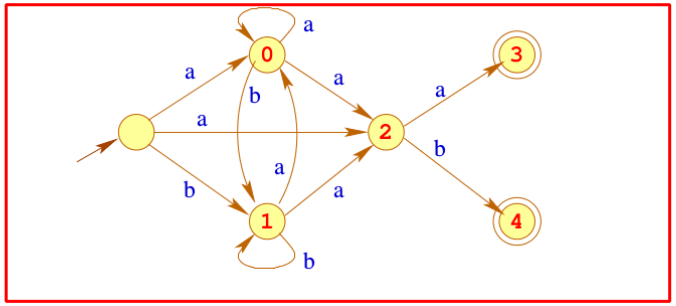
Final states: $\text{last}[e]$ if $\text{empty}[e] = f$
$\{\bullet e\} \cup \text{last}[e]$ otherwise

Transitions: $(\bullet e, a, i\bullet)$ if $i \in \text{first}[e]$ and i labled with a.
$(i\bullet, a, i'\bullet)$ if $i' \in \text{next}[i]$ and i' labled with a.

We call the resulting automaton A_e.

Berry-Sethi Approach

... for example:



Remarks:

- This construction is known as Berry-Sethi- or Glushkov-construction.
- It is used for XML to define Content Models
- The result may not be, what we had in mind...

Chapter 4:
Turning NFAs deterministic