Forward Declarations

Most programming languages admit the definition of recursive data types and/or recursive functions.

- A recursive definition needs to mention a name that is currently being defined or that will be defined later on.
- Old-fashioned programming languages require that these cycles are broken by a *forward* declaration.

Consider the declaration of an alternating linked list in C:

```c
struct list1 {
    int info;
    struct list0* next;
};
struct list0 {
    double info;
    struct list1* next;
};
```

```c
// the first declaration struct list1; is a forward declaration.
```
Declarations of Function Names

An analogous mechanism is need for (recursive) functions:

- in case a **recursive function** merely calls itself, it is sufficient to add the name of a function to the symbol table before visiting its body; example:

```c
int fac(int i) {
    return i * fac(i-1);
}
```

- for **mutually recursive functions** all function names at that level have to be entered (or declared as forward declaration).

Example: ML and C:

```c
fun odd 0 = false
| odd 1 = true
| odd x = even (x-1)
and even 0 = true
| even 1 = false
| even x = odd (x-1)
```

```c
int even(int x) {
    int odd(int x) {
        return (x==0 ? 0 :
                 (x==1 ? 1 : even(x-1)));
    }

    int even(int x) {
        return (x==0 ? 1 :
                 (x==1 ? 0 : odd(x-1)));
    }
}
```

Overloading of Names

The problem of using names before their declarations are visited is also common in object-oriented languages:

- for OO-languages with inheritance, a method's signature contributes to determining its binding
  - qualifies a function symbol with its parameters types
  - also requires resolution of parameter and return types
- the base class must be visited before the derived class in order to determine if declarations in the derived class are correct

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Once the types are resolved, other semantic analyses can be applied such as **type checking** or **type inference.**
Multiple Classes of Identifiers

Some programming languages distinguish between several classes of identifiers:
- C: variable names and type names
- Java: classes, methods, and fields
- Haskell: type names, constructors, variables, infix variables and -constructors

In some cases a declaration may change the class of an identifier; for example, a `typedef` in C:
- the scanner generates a different token, based on the class into which an identifier falls
- the parser informs the scanner as soon as it sees a declaration that changes the class of an identifier

the interaction between scanner and parser is problematic.
Type Synonyms and Variables in C

The C grammar distinguishes `typedef-name` and `identifier`. Consider the following declarations:

```
typedef struct { int x, y } point_t;
point_t origin;
```

Relevant grammar:
```
declarator → (declaration-specifier)^ declarator ;
declaration-specifier → static | volatile ... typedef | void | char | char ... typename
declarator → identifier | ...
```

**Problem:**
- parser adds `point_t` to the table of types when the `declaration` is reduced
- parser state has at least one look-ahead token

Type Synonyms and Variables in C: Solutions

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Type Synonyms and Variables in C: Solutions

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\[
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\text{declaration-specifier} & \rightarrow \text{static | volatile \ldots typedef} \\
& \quad | \text{void | char | char \ldots typedef} \\
\text{declarator} & \rightarrow \text{identifier | \ldots}
\end{align*}
\]

Solution is difficult:
- try to fix the look-ahead inside the parser

- add a rule to the grammar:

\[
\text{typename} \rightarrow \text{identifier}
\]

S/R- & R/R- Conflicts!!

Chapter 3:
Type Checking
Goal of Type Checking

In most mainstream (imperative / object oriented / functional) programming languages, variables and functions have a fixed type. For example: `int, void*, struct { int x; int y; }`.

Types are useful to:
- manage memory
- to avoid certain run-time errors

In imperative and object-oriented programming languages a declaration has to specify a type. The compiler then checks for a type correct use of the declared entity.

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Type Expressions

Types are given using type-expressions. The set of type expressions $T$ contains:
- base types: `int, char, float, void,...`
- type constructors that can be applied to other types
Type Expressions

Types are given using type-expressions. The set of type expressions $T$ contains:
- **base types**: int, char, float, void, ...
- **type constructors** that can be applied to other types

Example for type constructors in C:
- records: `struct { t_1 a_1; ... t_k a_k; }`
- pointer: `*` (pointer)
- arrays: `t [n]`
  - the size of an array can be specified
  - the variable to be declared is written between $t$ and $[n]$
- functions: `t (t_1, ..., t_k)`
  - the variable to be declared is written between $t$ and $(t_1, ..., t_k)$
  - in ML function types are written as: $t_1 * ... * t_k \rightarrow t$

Type Definitions in C

A type definition is a synonym for a type expression. In C they are introduced using the `typedef` keyword.

Type definitions are useful
- as abbreviation:
  ```c
typedef struct { int x; int y; } point_t;
```
- to construct recursive types:

Possible declaration in C:
```c
typedef struct list { list_t *next; } list_t;
```
```c
struct list {
  struct list *next;
  int info;
} list_t;
```
```c
struct list *head;
```
Type Checking using the Syntax Tree

Check the expression $a \{ f ( b \rightarrow c ) \} + 2$:

Idea:
- traverse the syntax tree bottom-up
- for each identifier, we lookup its type in $\Gamma$
- constants such as 2 or 0.5 have a fixed type
- the types of the inner nodes of the tree are deduced using typing rules

Type Systems

Formally: consider judgements of the form:

$$\Gamma \vdash e : t$$

// (in the type environment $\Gamma$ the expression $e$ has type $t$)

Axioms:

- **Const:** $\Gamma \vdash c : t_c$ (where $t_c$ is the type of constant $c$)
- **Var:** $\Gamma \vdash x : \Gamma(x)$ (where $x$ is a variable)

Rules:

Ref: $\Gamma \vdash e : t \quad \Gamma \vdash & e : t *$

Deref: $\Gamma \vdash * e : t$

Type Systems for C-like Languages

More rules for typing an expression:

- **Array:**
  $$\frac{\Gamma \vdash e_1 : t \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1[e_2] : t}$$

- **Array:**
  $$\frac{\Gamma \vdash e_1 : t \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1[e_2] : t}$$

- **Struct:**
  $$\frac{\Gamma \vdash e : \text{struct} \{ t_1 a_1 ; \ldots ; t_m a_m ; \} }{\Gamma \vdash e.a_i : t_i}$$

- **App:**
  $$\frac{\Gamma \vdash e : t ( t_1 , \ldots , t_m ) \quad \Gamma \vdash e_1 : t_1 \ldots \Gamma \vdash e_m : t_m}{\Gamma \vdash e ( e_1 , \ldots , e_m ) : t}$$

- **Op:**
  $$\frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 + e_2 : \text{int}}$$

- **Cast:**
  $$\frac{\Gamma \vdash e_1 : t_1 \quad \text{can be converted to } t_2}{\Gamma \vdash \langle t_1 \rangle e : t_2}$$

Example: Type Checking

Given expression $a \{ f ( b \rightarrow c ) \} + 2$ and $\Gamma = \{$

- **struct** list { int info; struct list next; };
- **int f ( struct list* l);**
- **struct** { struct list* c1; by
  int a [ 11 ];

- $\}.$

Cast: $\frac{\Gamma \vdash e : t_1 \quad \text{can be converted to } t_2}{\Gamma \vdash \langle t_1 \rangle e : t_2}$
Type Systems

Formally: consider judgements of the form:
\[
\Gamma \vdash e : t
\]

// (in the type environment \( \Gamma \), the expression \( e \) has type \( t \))

Axioms:
- Const: \( \Gamma \vdash c : t_c \) (\( t_c \) type of constant \( c \))
- Var: \( \Gamma \vdash x : \Gamma(x) \) (\( x \) Variable)

Rules:
- Ref: \( \frac{\Gamma \vdash e : t}{\Gamma \vdash \& e : t} \)
- Deref: \( \frac{\Gamma \vdash \& e : t}{\Gamma \vdash \ast e : t} \)

Type Systems for C-like Languages

More rules for typing an expression:

- Array:
  \[
  \frac{\Gamma \vdash e_1 : t \ast \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1[e_2] : t}
  \]
- Array:
  \[
  \frac{\Gamma \vdash e_1 : t \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1[e_2] : t}
  \]

- Struct:
  \[
  \frac{\Gamma \vdash e : \text{struct} \{t_1, a_1; \ldots ; t_m, a_m;\}}{\Gamma \vdash e[a_i] : t_i}
  \]

- App:
  \[
  \frac{\Gamma \vdash e : t(t_1, \ldots , t_m) \quad \Gamma \vdash e_1 : t_1 \ldots \Gamma \vdash e_m : t_m}{\Gamma \vdash e(e_1, \ldots , e_m) : t}
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- Op:
  \[
  \frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 + e_2 : \text{int}}
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- Cast:
  \[
  \frac{\Gamma \vdash e : t_1 \quad \Gamma \vdash (t_2) e : t_2 \text{ can be converted to } t_2}{\Gamma \vdash e : t_2}
  \]

Example: Type Checking

Given expression `*a[f(b->c)]+2` and \( \Gamma = \{
\text{struct list \{ int info; struct list* next; \};}
\text{int f(struct list* l);}\]
\text{struct \{ struct list* c; \} = b;}
\text{int a[i][i];}
\}\):

- Array:
  \[
  \frac{\Gamma \vdash e_1 : t \ast \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1[e_2] : t}
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  \[
  \frac{\Gamma \vdash e : \text{struct} \{t_1, a_1; \ldots ; t_m, a_m;\}}{\Gamma \vdash e[a_i] : t_i}
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- App:
  \[
  \frac{\Gamma \vdash e : t(t_1, \ldots , t_m) \quad \Gamma \vdash e_1 : t_1 \ldots \Gamma \vdash e_m : t_m}{\Gamma \vdash e(e_1, \ldots , e_m) : t}
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  \[
  \frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 + e_2 : \text{int}}
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  \[
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  \]
Equality of Types

Summary type checking:
- Choosing which rule to apply at an AST node is determined by the type of the child nodes
- \( \sim \) determining the rule requires a check for equality of types

Type equality in C:
- `struct A { }` and `struct B { }` are considered to be different
  - the compiler could re-order the fields of A and B independently (not allowed in C)
  - to extend an record A with more fields, it has to be embedded into another record:
    ```
    typedef struct B {
      struct A a;
      int field_of_B;
    } extension_of_A;
    ```
  - after issuing `typedef int C;` the types C and int are the same

Algorithm for Testing Structural Equality

Idea:
- track a set of equivalence queries of type expressions
- if two types are syntactically equal, we stop and report success
- otherwise, reduce the equivalence query to a several equivalence queries on (hopefully) simpler type expressions

Suppose that recursive types were introduced using type equalities of the form:

\[
A = t
\]

(we omit the \( \Gamma \)). Then define the following rules:

Structural Type Equality

Alternative interpretation of type equality (does not hold in C):

semantically, two type \( t_1, t_2 \) can be considered as equal if they accept the same set of access paths.

Example:

```c
struct list {    struct list1 {
  int info;
  struct list* next;
}    struct list1 { 
  int info;
  struct list1* next;
}    struct list1* next;
}
```

Consider declarations `struct list* l` and `struct list1* l`. Both allow

\[
l->info \quad l->next->info
\]

but the two declarations of l have unequal types in C.

Rules for Well-Typedness
Example:

\[
A = \text{struct \{int info; A* next; \}} \\
B = \text{struct \{int info; \} \ast \text{next; \}}
\]

We ask, for instance, if the following equality holds:

\[
\text{struct \{int info; A* next; \}} = B
\]

We construct the following deduction tree:

![Deduction Tree Image]

Proof for the Example:

\[
A = \text{struct \{int info; A* next; \}} \\
B = \text{struct \{int info; \} \ast \text{next; \}}
\]

\[
\text{struct \{int info; A* next; \}} = B
\]

![Proof Diagram Image]

Implementation

We implement a function that implements the equivalence query for two types by applying the deduction rules:

- if no deduction rule applies, then the two types are not equal
- if the deduction rule for expanding a type definition applies, the function is called recursively with a potentially larger type
- during the construction of the proof tree, an equivalence query might occur several times
- in case an equivalence query appears a second time, the types are by definition equal

Termination?

- the set \( D \) of all declared types is finite
- there are no more than \( |D|^3 \) different equivalence queries
- repeated queries for the same inputs are are automatically satisfied
- termination is ensured

Overloading and Coercion

Some operators such as + are overloaded:

- + has several possible types
  - for example: int + (int, int), float + (float, float)
  - but also float* + (float*, int), int* + (int, int*)
- depending on the type, the operator + has a different implementation
- determining which implementation should be used is based on the arguments only